JAYPEE UNIVERSITY OF INFORMATION TECHNOLOGY, WAKNAGHAT

TEST -1 EXAMINATION- 2023

M.Tech-I Semester (CS: Data Science)

COURSE CODE (CREDITS): 22M11MA111 (3)　　　　　　MAX. MARKS: 15

COURSE NAME: Mathematical Foundations for Data Science

COURSE INSTRUCTORS: Saurabh Srivastava　　　　　　MAX. TIME: 1 Hour

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Note: **(a)** *All questions are compulsory.*

(b) *Marks are indicated against each question in square brackets.*

(c) *The candidate is allowed to make Suitable numeric assumptions wherever required for solving problems.*

(d) *Use of scientific calculator is allowed.*

Q.1 Find the directional derivative of $xe^x + \cos(xy)$ at the point $\left(-\frac{\pi}{2}, 1\right)$ in the direction of the vector 2i- 5j.　　　　　　**[3 M] (CO-1)**

Q.2 Find the directions in which the function $\frac{x}{y} - yz$ increases or decreases most rapidly at the point (4,1,1), then find the derivative in this direction also.　　　　　　**[3 M] (CO-1)**

Q.3 Find the extreme values of $3x - y + 6$ subject to the constraints $x^2 + y^2 = 4$.

[3 M] (CO-1)

Q.4 For the matrix $\begin{bmatrix} 1 & -2 \\ -5 & 4 \end{bmatrix}$, find

　　a)　singular values of the given matrix.

　　b)　find it's singular value decomposition.　　　　　　**[4 M] (CO-2)**

Q.5 Prove that the set of all real valued functions is a vector space over the field of the real numbers with the following operations:

$(f + g)(x) = f(x) + g(x)$, and $(kf)(x) = kf(x), k \in \mathbb{R}, x \in \mathbb{R}$.　　　　　　**[2 M] (CO-2)**