

Jaypee University of Information Technology, Wagnaghat

Test-2 Examinations, October 2023

B.Tech - VII Semester (ALL)

Course Code/Credits: 22B1WMA731/3

Course Title: Linear Algebra for Machine Learning & Data Science

Course Instructor: RAD

Max. Marks: 25

Max. Time: 90 mins

**Instructions:** All questions are compulsory. Marks are indicated against each question.

1. Answer the following with justification: (4 Marks) [CO-1]

(a) Is the set of all  $3 \times 3$  matrices  $A$  such that  $\det(A) = 1$  a subspace  $\mathcal{M}_{3 \times 3}(\mathbb{R})$  of  $3 \times 3$  matrices over  $\mathbb{R}$ ?

(b) Is the set of all vectors of the form  $(a, b, 0) \in \mathbb{R}^3$  with  $b = 3a$  a subspace of  $\mathbb{R}^3$ ?

2. Consider the following vectors: (4 Marks) [CO-1]

$$\mathbf{v}_1 = \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix}; \quad \mathbf{v}_2 = \begin{pmatrix} 0 \\ -5 \\ 1 \end{pmatrix}; \quad \mathbf{v}_3 = \begin{pmatrix} 6 \\ -7 \\ -9 \end{pmatrix}$$

Determine whether these vectors span  $\mathbb{R}^3$ ?

3. Consider the following  $3 \times 3$  matrix: (4 Marks) [CO-3]

$$\mathbf{B} = \begin{bmatrix} 0 & 1 & 3 \\ -1 & 0 & 1 \\ -1 & 2 & 7 \end{bmatrix}$$

(a) Find the *column space* of the matrix  $\mathbf{B}$ .

(b) Determine a *basis* for the *Columnspace*( $\mathbf{B}$ ).

4. Consider the following subspace of  $\mathbb{R}^3$ : (4 Marks) [CO-2]

$$\mathbf{C} = \text{Span} \left\{ \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \right\}$$

(a) Determine the *orthogonal complement* of  $\mathbf{C}$ .

(b) What is the dimension of  $\mathbf{C}^\perp$ ?

5. Let  $\mathbf{T} : \mathbb{R}^3 \rightarrow \mathbb{R}^3$  be the *projection*  $\mathbf{T}(\mathbf{u}) = \text{Proj}_{\mathbf{v}}(\mathbf{u})$  for  $\mathbf{v} = (1, 1, 1)$ . (4 Marks) [CO-3]

(a) Find  $\mathbf{T}(x, y, z)$ .

(b) Compute  $\mathbf{T}(5, 0, 5)$ .

(c) Determine the standard matrix of  $\mathbf{T}$ .

6. Consider the following *over-determined* system  $\mathbf{Ax} = \mathbf{b}$  for  $\mathbf{x} \in \mathbb{R}^2$ : (5 Marks) [CO-3]

$$\mathbf{A} = \begin{bmatrix} 1 & 1 \\ 10^{-10} & 0 \\ 0 & 10^{-10} \end{bmatrix}; \quad \mathbf{b} = \begin{bmatrix} 1 \\ 10^{-10} \\ 10^{-10} \end{bmatrix}$$

- (a) Find the *orthogonal projection* of  $\mathbf{b}$  onto the columns of  $\mathbf{A}$ .
- (b) Determine the *least-squares* solution of  $\mathbf{Ax} = \mathbf{b}$ .

\* \* \* \* \*