JAYPEE UNIVERSITY OF INFORMATION TECHNOLOGY, WAKNAGHAT

TEST -2 EXAMINATION- 2023

B. Sc.- I Semester (Mathematics and Computing)

COURSE CODE (CREDITS): 22BS1MA111 (04) MAX. MARKS: 25

COURSE NAME: Calculus

COURSE INSTRUCTOR: Prof. K Singh MAX. TIME: 1 Hour 30 Minutes

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Note: *(a) All questions are compulsory.*

(b) Marks are indicated against each question in square brackets.

(c) The candidate is allowed to make Suitable numeric assumptions wherever required for solving problems

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Q.1. Find the radius of curvature for the space curve

$$\bar{r}(t) = (e^t cost)i + (e^t sint)j + 2k.$$ [CO-3] [4]

Q.2. Find parametric equations and a parameter interval for the motion of a particle that starts at $(a, 0)$ and traces the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ once clockwise. [CO-2] [4]

Q.3. Show that $sech^{-1}x = ln\left(\frac{1+\sqrt{1-x^2}}{x}\right),\ 0 < x \leq 1.$ [CO-2] [5]

Q.4. Use the fact that $\frac{d}{dx}(3x^4 + x^2 - 4x) = 12x^3 + 2x - 4$ to show that the equation $12x^3 + 2x - 4 = 0$ has at least one solution in the interval $(0, 1)$. [CO-3] [4]

Q.5. Find the x-coordinate of the point on the graph of $y = x^2$ where the tangent line is parallel to the secant line that cuts the curve at $x = -1$ and $x = 2$. [CO-2] [4]

Q.6. Find the length of the arc of the curve $y = x^{3/2}$ on $[0, 4]$. [CO-3] [4]

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TEST -2 EXAMINATION- 2023

M.Sc-I Semester (BT/MB)

COURSE CODE (CREDITS):20MS1MA111(02) MAX. MARKS: 25

COURSE NAME: Basics of Mathematics and Statistics

COURSE INSTRUCTOR:Dr.Neel Kanth MAX. TIME: 1 Hour 30 Minutes

Note: *(a)All questions are compulsory.*

(b)Marks are indicated against each question in square brackets.

(c) The candidate is allowed to make Suitable numeric assumptions wherever required for solving problems

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Q1.If $A = \begin{bmatrix} 3 & 2 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 5 \end{bmatrix}$,show that $A^2 - 7A + 10I_3$ is a null matrix. [5]

Q2.For two matrices $A = \begin{bmatrix} 2 & 1 & 3 \\ 4 & 1 & 0 \end{bmatrix}$ $and\ B = \begin{bmatrix} 1 & -1 \\ 0 & 2 \\ 5 & 0 \end{bmatrix}$ verify that $(AB)^T = B^T A^T$ [5]

Q3.Solve the linear system of equations using Cramer's rule [5]

$$x + y + z = 8, \quad 4x + 2y + z = 11 \ and \ 9x - 3y + z = 6$$

Q4. Simplify and find the result in the form $a + ib$ [4]

a) $\left(\frac{3+2i}{2-3i}\right) + \left(\frac{3-2i}{2+3i}\right)$

b) $\frac{(2+3i)^2}{2-i}$

Q5.Find the least positive value of n, if $\left(\frac{1+i}{1-i}\right)^n = 1$ [2]

Q6.If $z_1 = 2 - i \ and \ z_2 = 1 + i$, find $\left|\frac{z_1+z_2+1}{z_1-z_2+i}\right|$ [4]

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TEST -2 EXAMINATION- 2023

B.Tech-I Semester (B.Sc)

COURSE CODE (CREDITS):22BS1MA112 (04) MAX. MARKS: 25

COURSE NAME: LINEAR ALGEBRA

COURSE INSTRUCTORS: BKP*, MDS MAX. TIME: 1 Hour30 Minutes

Note: (a)All questions are compulsory.

(b)Marks are indicated against each question in square brackets.

(c) The candidate is allowed to make Suitable numeric assumptions wherever required for solving problems

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Q.1 For what value of λ the following system of equation has unique solution. Also find the solution in this case. (CO-1) [5]

$$3x - y + 4z = 3,\ x + 2y - 3z = -2,\ 6x + 5y + \lambda z = -3$$

Q.2 Let V be the set of all ordered pairs (x, y) of real numbers, and let F be the field of real numbers. Define (CO-3) [4]

$$(x,\ y) + (x_1,\ y_1) = (x + x_1, y + y_1),\quad \forall\ (x,y), (x_1, y_1) \in V$$
$$c(x,y) = (|c|x, |c|y),\quad \forall\ c \in F.$$

Is V, with these operations, a vector space over the field of real number? Justify your answer?

Q.3 Let $v_1 = (1,\ -1,\ 0)$, $v_2 = (0,\ 1,\ -1)$, $v_3 = (0,\ 2,\ 1)$ and $v_4 = (1, 0, 3)$ be elements of $\boldsymbol{R^3}$. Show that the set of vectors $\{v_1, v_2, v_3, v_4\}$ is linearly dependent. (CO-3) [4]

Q.4 Suppose $T: R^3 \to R^2$ is a linear transformation with $T(1, -1,\ 0) = (2, 1)$, $T(0,\ 1, -1) = (-1,\ 3)$ and $T(0,1,0) = (0, 1)$, where $\{(1, -1,\ 0), (0,\ 1, -1),\ (0,1,0)\}$ form a basis of $\boldsymbol{R^3}$. Then find

a) The formula for $T(x, y, z)$, for any $(x, y, z) \in R^3$.
b) Null space and Nullity of T
c) Range space and rank of T

(CO-3) [3+2+2]

Q.5 Let $T: R^3 \to R^2$ be the linear transformation given by $T(x, y, z) = (x + y,\ y - z)$. Let $B_1 = \{v_1, v_2, v_3\}$ and $B_2 = \{w_1, w_2\}$ be bases for R^3 and R^2, respectively, where $v_1 = (1,\ 0, 1)$, $v_2 = (0,\ 1,\ 1)$, $v_3 = (1,1,1)$, $w_1 = (1,\ 2)$ and $w_2 = (-1,\ 1)$. Find the matrix of T with respect to B_1 and B_2.

(CO-3) [5]

JAYPEE UNIVERSITY OF INFORMATION TECHNOLOGY, WAKNAGHAT

TEST -2 EXAMINATION- 2023

Ph.D.-I Semester (Mathematics)

COURSE CODE (CREDITS): 17P1WMA231 (3) MAX. MARKS: 25

COURSE NAME: ADVANCED LINEAR ALGEBRA

COURSE INSTRUCTOR: Pradeep Kumar Pandey MAX. TIME: 1 Hour 30 Minutes

***Note: (a)** All questions are compulsory.*

(b) Marks are indicated against each question in square brackets.

(c) The candidate is allowed to make Suitable numeric assumptions wherever required for solving problems.

1. For any $u, v \in \mathbb{R}^+$ and $\alpha \in \mathbb{R}$, consider the operations $\oplus$ and $\odot$ defined as: $u \oplus v = uv$ and $\alpha \odot u = u^\alpha$. Prove or disprove that the algebraic structure $(\mathbb{R}^+, \oplus, \odot)$ is a vector space over the Field $\mathbb{R}$. [CO-1] [5M]

2. State the Cayley-Hamilton theorem, and using it reduce the degree of polynomial $A^3 - 5A^2 + 2I$ for the matrix $A = \begin{bmatrix} 3 & 7 \\ 0 & 1 \end{bmatrix}$. [CO-1] [5M]

3. Suppose $B = \{(1,0,1), (1,-1,0), (2,0,-1)\}$ is an ordered basis of $\mathbb{R}^3$. Find the dual basis of B. [CO-2] [5M]

4. Check consistency of the following linear system, and solve it by method of least squares

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \end{bmatrix}^T \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 & 3 & 4 & 4 \end{bmatrix}^T$$ [CO-3] [5M]

5. Using Gram-Schmidt method on the columns of matrix A find its QR decomposition

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$ [CO-3] [5M]

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TEST-2 EXAMINATION- 2023

B.Tech-I Semester (BT/BI)

COURSE CODE (CREDITS): 18B11MA112 (04) MAX. MARKS: 25

COURSE NAME: BASIC MATHEMATICS-I

COURSE INSTRUCTORS: MDS MAX. TIME: 1 Hour 30 Minutes

Note: *(a)All questions are compulsory.*

(b)Marks are indicated against each question in square brackets.

(c) The candidate is allowed to make Suitable numeric assumptions wherever required for solving problems

Q.1 For what values of x, the matrix $\begin{bmatrix} 3-x & 2 & 2 \\ 2 & 4-x & 1 \\ -2 & -4 & -1-x \end{bmatrix}$ is singular? **(CO-1) [4]**

Q.2 If $A = \begin{bmatrix} 1 & -2 & 3 \\ 2 & 3 & -1 \\ -3 & 1 & 2 \end{bmatrix}$, and I is the identity matrix of order 3, evaluate $A^2 - 3A + 9I$. **(CO-1) [4]**

Q.3 Find the vector projection of a force $F = 5\hat{\imath} + 4\hat{\jmath} + \hat{k}$ onto $\vec{V} = 3\hat{\imath} + 5\hat{\jmath} - 2\hat{k}$ and the scalar component of $\vec{F}$ in the direction of $\vec{V}$. **(CO-2) [3]**

Q.4 **(a)** Find the vector, parametric and Cartesian equations for the line through $(-3, 2, -3)$ and $(1, -1, 4)$.

(b) Find the vector equations of the plane which is at a distance of 5 units from the origin and which is normal to the vector $4\hat{\imath} - 3\hat{\jmath} + 5\hat{k}$. **(CO-2) [3+2]**

Q.5 Find the shortest distance between the lines **(CO-2) [5]**

$$\vec{r} = (3\hat{\imath} + 4\hat{\jmath} - 2\hat{k}) + \lambda(-\hat{\imath} + 2\hat{\jmath} + \hat{k})$$

and

$$\vec{r} = (\hat{\imath} - 7\hat{\jmath} - 2\hat{k}) + \mu(\hat{\imath} + 3\hat{\jmath} + 2\hat{k}).$$

Q.6 Find the real value of x and y if **(CO-3) [4]**

$$\frac{1}{x+iy} - \frac{1}{1+i} = 2 - 3i$$

JAYPEE UNIVERSITY OF INFORMATION TECHNOLOGY, WAKNAGHAT

TEST -2 EXAMINATION- 2023

B.Tech-III Semester (CE)

COURSE CODE (CREDITS): 18B11MA311 (3) MAX. MARKS: 25

COURSE NAME: NUMERICAL METHODS

COURSE INSTRUCTOR: Pradeep Kumar Pandey MAX. TIME: 1 Hour 30 Minutes

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***Note:** (a) All questions are compulsory.*

(b) Marks are indicated against each question in square brackets.

(c) The candidate is allowed to make Suitable numeric assumptions wherever required for solving problems.

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1. Using power method, obtain the dominant eigenvalue of the following matrix:

$$\begin{bmatrix} 4 & 2 & -2 \\ -2 & 8 & 1 \\ 2 & 4 & -4 \end{bmatrix}$$

Take initial eigenvector $X_0 = [1 \quad 1 \quad 1]^T$. Write your answer correct to 3 decimal places and up to 4th iteration. [CO2] [5M]

2. Solve the following system of equations by Doolittle's (LU decomposition) method:
$x + 5y + z = 14,\ 2x + y + 3z = 13,\ 3x + y + 4z = 17.$ [CO2] [5M]

3. Obtain the Lagrange's interpolating polynomial for the following data:

x_i	-1	0	1
y_i	3	8	11

and, using so-obtained Lagrange's interpolating polynomial, find approximate value of the function at $x = 0.4$. [CO3] [5M]

4. Construct the divided difference table, and using Newton's divided difference formula, obtain the interpolating polynomial for the following data: [CO3] [5M]

x_i	0	1	3	4
f_i	-5	1	25	55

5. Use the method of least squares to fit a straight line to the data given below: [CO4] [5M]

x_i	5	10	15	20	25
y_i	15	19	23	26	30

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JAYPEE UNIVERSITY OF INFORMATION TECHNOLOGY, WAKNAGHAT

TEST -2 EXAMINATION- 2023

B.Tech-III Semester (CE)

COURSE CODE (CREDITS): 18B11MA311 (3) MAX. MARKS: 25

COURSE NAME: NUMERICAL METHODS

COURSE INSTRUCTOR: Pradeep Kumar Pandey MAX. TIME: 1 Hour 30 Minutes

Note: *(a) All questions are compulsory.*

(b) Marks are indicated against each question in square brackets.

(c) The candidate is allowed to make Suitable numeric assumptions wherever required for solving problems.

1. Using power method, obtain the dominant eigenvalue of the following matrix:

$$\begin{bmatrix} 4 & 2 & -2 \\ -2 & 8 & 1 \\ 2 & 4 & -4 \end{bmatrix}$$

Take initial eigenvector $X_0 = [1 \quad 1 \quad 1]^T$. Write your answer correct to 3 decimal places and up to 4th iteration. [CO2] [5M]

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x_i	5	10	15	20	25
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JAYPEE UNIVERSITY OF INFORMATION TECHNOLOGY, WAKNAGHAT

T-2, EXAMINATION- 2023

B. Tech. I Semester (CSE/IT/ECE/CE)

COURSE CODE (CREDITS): 18B11MA111 (04) MAX. MARKS: 25

COURSE NAME: ENGINEERING MATHEMATICS-I

COURSE INSTRUCTORS: RKB, KAS, NKT, BKP, PKP, MDS*, MAX. TIME: 1 Hour 30 Minutes.

Note: *(a) All questions are compulsory.*

(b)Marks are indicated against each question in square brackets.

(c) The candidate is allowed to make Suitable numeric assumptions wherever required for solving problems

(d) Use of scientific calculator is not allowed.

Q.1 If $= (1-2xy+y^2)^{-1/2}$, Show that $x\frac{\partial u}{\partial x} - y\frac{\partial u}{\partial y} = y^2 u^3$. [2] [CO-1]

Q.2 Find the points on the surface $z^2 = xy + 1$ nearest to the origin. [3] [CO-2]

Q.3 (a) Is the area under the curve $y = e^{-\sqrt{x}}$ from $x = 0$ to $x = \infty$ finite? If so, what is its value? [3+3] [CO-3]

(b) Show that

$$\int_0^a x^2 (a^2 - x^2)^{3/2} dx = \frac{\pi a^6}{32}.$$

Q.4 Draw a rough sketch of the region of integration of

$$\int_{y=0}^{y=4} \int_{x=y}^{x=4} \frac{x}{x^2+y^2} dx\, dy,$$

and hence evaluate it by changing its order of integration. [5] [CO-3]

Q.5 Find the directions in which the function $f(x,y,z) = \frac{x}{y} - yz$ increases and decreases most rapidly at the point $P(4,1,1)$. Also, find the derivatives in these directions. [4] [CO-4]

Q.6 (a) Find the tangent vector and equation of the tangent line to the curve whose parametric representation is [2+3] [CO-4]

$$x = 2t^2;\; y = t;\; z = 3t^3 \text{ at } t = 2.$$

(b) Find the equation of normal line to the surface $z = -x^2 - y^2 + 2$ at $(0,1,1)$.