JAYPEE UNIVERSITY OF INFORMATION TECHNOLOGY, WAKNAGHAT

TEST -2 EXAMINATION- 2023

M.Tech.-I Semester (CSE: Data Science)

COURSE CODE (CREDITS): 22M11MA111 (3) MAX. MARKS: 25

COURSE NAME: Mathematical Foundations for Data Science

COURSE INSTRUCTOR: Saurabh Srivastava MAX. TIME: 1 Hour 30 Minutes

---

Note: *(a) All questions are compulsory.*

(b) Marks are indicated against each question in square brackets.

(c) The candidate is allowed to make suitable numeric assumptions wherever required for solving problems.

(d) Use of scientific calculator is allowed.

---

1. An internet service provider charges its customers for the time of the internet use rounding it up to the nearest hour. The joint distribution of the used time X (hours) and the charge per hour Y (cents) is given in the table below.

$p_{XY}(x,y)$		x			
		1	2	3	4
y	1	0	0.06	0.06	0.10
	2	0.10	0.10	0.04	0.04
	3	0.40	0.10	0	0

a) Find the marginal probability mass function of X.

b) Find the conditional probability of $Y = 3$, given X= 2.

c) Find the mathematical expectation of X.

d) Find the covariance of X and Y.

e) Are X and Y independent? **[5M](CO-3)**

2. A software package consists of 12 programs, five of which must be upgraded. If 4 programs are randomly chosen for testing,

a) What is the probability that at least two of them must be upgraded?

b) What is the expected number of programs, out of the chosen four, that must be upgraded? **[2M+2M](CO-3)**

3. The article "Reliability of Domestic-Waste Biofilm Reactors" (J. of Envir. Engr., 1995: 785–790) suggests that substrate concentration of influent to a reactor is normally distributed with mean 0.30 and standard deviation 0.06.

a) What is the probability that the concentration exceeds 0.25?

b) What is the probability that the concentration is at most 0.42? **[2M+2M](CO-3)**

4. Suppose pulses arrive at a counter at an average rate of six per minute, find the probability that in a 0.5-min interval:
   a) At least one pulse is received.
   b) Exactly five pulses are received. **[2M+2M](CO-3)**

5. Fuses produced by a company will blow in 12.40 minutes on the average when overloaded. Suppose the mean blow time of 20 fuses subjected to overload is 10.63 minutes and standard deviation 2.48 minutes. Does this information tend to support or refute the claim that the population mean blow time is 12.40 minutes? **[4M](CO-3)**

6. Each front tire on a particular type of vehicle is supposed to be filled to a pressure of 26 psi. Suppose the actual air pressure in each tire is a random variable X for the right tire and Y for the left tire, with joint probability density function:

$$f(x,y) = \begin{cases} K(x^2+y^2), 20 \le x \le 30, 20 \le y \le 30 \\ 0, \text{ else} \end{cases}$$

   a) What is the value of K?
   b) What is the probability that both tires are under filled? **[2M+2M](CO-3)**

Standard Normal Values:

0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817

t-Table:

	α						
v	0.40	0.30	0.20	0.15	0.10	0.05	0.025
16	0.258	0.535	0.865	1.071	1.337	1.746	2.120
17	0.257	0.534	0.863	1.069	1.333	1.740	2.110
18	0.257	0.534	0.862	1.067	1.330	1.734	2.101
19	0.257	0.533	0.861	1.066	1.328	1.729	2.093