COURSE CODE (CREDITS): 18B11MA111 (04)

COURSE NAME: ENGINEERING MATHEMATICS-I

COURSE INSTRUCTORS: RKB, KAS, NKT, BKP, PKP, MDS*,

MAX. MARKS: 25

MAX. TIME: 1 Hour 30 Minutes.

Note: (a) *All questions are compulsory.*

(b)Marks are indicated against each question in square brackets.

(c) The candidate is allowed to make Suitable numeric assumptions wherever required for solving problems

(d) Use of scientific calculator is not allowed.

Q.1 If $= (1 - 2xy + y^2)^{-1/2}$, Show that $x\frac{\partial u}{\partial x} - y\frac{\partial u}{\partial y} = y^2 u^3$. [2] [CO-1]

Q.2 Find the points on the surface $z^2 = xy + 1$ nearest to the origin. [3] [CO-2]

Q.3 (a) Is the area under the curve $y = e^{-\sqrt{x}}$ from $x = 0$ to $x = \infty$ finite? If so, what is its value?

(b) Show that [3+3] [CO-3]

$$\int_0^a x^2(a^2 - x^2)^{3/2}dx = \frac{\pi a^6}{32}.$$

Q.4 Draw a rough sketch of the region of integration of

$$\int_{y=0}^{y=4} \int_{x=y}^{x=4} \frac{x}{x^2 + y^2} dx\, dy,$$

and hence evaluate it by changing its order of integration. [5] [CO-3]

Q.5 Find the directions in which the function $f(x,y,z) = \frac{x}{y} - yz$ increases and decreases most rapidly at the point $P(4, 1, 1)$. Also, find the derivatives in these directions. [4] [CO-4]

Q.6 (a) Find the tangent vector and equation of the tangent line to the curve whose parametric representation is [2+3] [CO-4]

$$x = 2t^2; \quad y = t; \quad z = 3t^3 \text{ at } t = 2.$$

(b) Find the equation of normal line to the surface $z = -x^2 - y^2 + 2$ at $(0,1,1)$.