JAYPEE UNIVERSITY OF INFORMATION TECHNOLOGY, WAKNAGHAT

TEST -2 EXAMINATION- 2023

B.Tech-I Semester (B.Sc)

COURSE CODE (CREDITS):22BS1MA112 (04) MAX. MARKS: 25

COURSE NAME: LINEAR ALGEBRA

COURSE INSTRUCTORS: BKP*, MDS MAX. TIME: 1 Hour30 Minutes

Note: *(a)All questions are compulsory.*

(b)Marks are indicated against each question in square brackets.

(c) The candidate is allowed to make Suitable numeric assumptions wherever required for solving problems

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Q.1 For what value of λ the following system of equation has unique solution. Also find the solution in this case. (CO-1) [5]

$$3x - y + 4z = 3,\ x + 2y - 3z = -2,\ 6x + 5y + \lambda z = -3$$

Q.2 Let V be the set of all ordered pairs (x, y) of real numbers, and let F be the field of real numbers. Define (CO-3) [4]

$$(x,\ y) + (x_1,\ y_1) = (x + x_1, y + y_1), \quad \forall\, (x, y), (x_1, y_1) \in V$$
$$c(x, y) = (|c|x, |c|y), \quad \forall\, c \in F.$$

Is V, with these operations, a vector space over the field of real number? Justify your answer?

Q.3 Let $v_1 = (1,\ -1,\ 0)$, $v_2 = (0,\ 1,\ -1)$, $v_3 = (0,\ 2,\ 1)$ and $v_4 = (1, 0, 3)$ be elements of $\boldsymbol{R}^3$. Show that the set of vectors $\{v_1, v_2, v_3, v_4\}$ is linearly dependent. (CO-3) [4]

Q.4 Suppose $T: R^3 \to R^2$ is a linear transformation with $T(1, -1,\ 0) = (2, 1)$, $T(0,\ 1, -1) = (-1,\ 3)$ and $T(0, 1, 0) = (0, 1)$, where $\{(1, -1,\ 0), (0,\ 1, -1),\ (0, 1, 0)\}$ form a basis of $\boldsymbol{R}^3$. Then find

a) The formula for $T(x, y, z)$, for any $(x, y, z) \in R^3$.
b) Null space and Nullity of T
c) Range space and rank of T

(CO-3) [3+2+2]

Q.5 Let $T: R^3 \to R^2$ be the linear transformation given by $T(x, y, z) = (x + y,\ y - z)$. Let $B_1 = \{v_1, v_2, v_3\}$ and $B_2 = \{w_1, w_2\}$ be bases for R^3 and R^2, respectively, where $v_1 = (1,\ 0, 1)$, $v_2 = (0,\ 1,\ 1)$, $v_3 = (1,1,1)$, $w_1 = (1,\ 2)$ and $w_2 = (-1,\ 1)$. Find the matrix of T with respect to B_1 and B_2.

(CO-3) [5]