Jaypee University of Information Technology, Waknaghat

Test-3 Examination, December 2023

B.Tech - VII Semester (ALL)

Course Code/Credits: 22B1WMA731/3 Max. Marks: 35
Course Title: Linear Algebra for Data Science & Machine Learning
Course Instructor: RAD Max. Time: 2 hours

Instructions: All questions are compulsory. Marks are indicated against each question.
Use of scientific calculators is allowed.

1. Consider the set of matrices: (4 Marks) [CO-1]

$$\mathbf{H} = \left\{ \begin{bmatrix} 2a & b \\ 3a+b & 3b \end{bmatrix} \; : \; a, b \in \mathbb{R} \right\}$$

   (a) Is $\mathbf{H}$ a subspace of set $\mathcal{M}_{2\times 2}(\mathbb{R})$ of all square matrices of order 2?
   (b) Justify your answer.

2. Consider the linear transformation $\mathbf{T}$ from $\mathcal{M}_{2\times 3}(\mathbb{R})$ to $\mathcal{M}_{2\times 2}(\mathbb{R})$: (4 Marks) [CO-4]

$$\mathbf{T}\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{pmatrix} = \begin{pmatrix} 2a_{11} - a_{12} & a_{13} + 2a_{12} \\ 0 & 0 \end{pmatrix}$$

   (a) Determine the kernel of the transformation $\mathbf{T}$.
   (b) Write down the basis for $\mathtt{Ker}(\mathbf{T})$.

3. Let $\mathbf{V}$ be the subspace spanned by $(1,1,0,1)$ and $(0,0,1,0)$. (5 Marks) [CO-4]

   (a) Determine the *orthogonal complement* of $\mathbf{V}$.
   (b) What is the dimension of $\mathbf{V}^{\perp}$.

4. Consider the *over-determined* system given by $\mathbf{Ax} = \mathbf{b}$: (5 Marks) [CO-3]

$$\begin{aligned} x_1 - x_2 &= 4 \\ 3x_1 + 2x_2 &= 1 \\ -2x_1 + 4x_2 &= 3 \end{aligned}$$

   (a) Determine the *least-squares* solution $\mathbf{x} \in \mathbb{R}^2$.
   (b) Find the *orthogonal projection* $\mathbf{b}$ on the column space of $\mathbf{A}$.

5. Consider the following 3×3 matrix: (5 Marks) [CO-3]

$$\mathbf{A} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

   (a) Apply Gram-Schmidt orthogonalization process to the column vectors of $\mathbf{A}$.
   (b) Find the QR-decomposition of $\mathbf{A}$.

6. Let **B** be the given 3×2 matrix: (6 Marks) [CO-3]

$$\begin{bmatrix} 7 & 1 \\ 0 & 0 \\ 5 & 5 \end{bmatrix}$$

(a) Determine the *singular values* of **B**.

(b) Compute *singular value decomposition* (SVD) of **B**.

7. Consider the following blood pressure measurements from 3 adults:

Features	Adult 1	Adult 2	Adult 3
Systolic BP	x_{11}	x_{12}	x_{13}
Diastolic BP	x_{21}	x_{22}	x_{23}

Correlation values obtained among the two variables in the dataset are given in the matrix

$$\mathbf{S} = \begin{bmatrix} 5 & 2 \\ 2 & 2 \end{bmatrix}.$$

Perform *principal component analysis* on the given dataset to answer: (6 Marks) [CO-4]

(a) Find the *eigenvalues* of **S**.

(b) Obtain the eigenvectors corresponding to the *principal components* of the dataset.

(c) What fraction of variation is explained by the first *principal component*?

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