Jaypee University of Information Technology, Waknaghat

Test-3 Examinations - December 2023

B.Tech - III Semester (CSE/IT)

Course Code/Credits: 18B11MA313/3 Max. Marks: 35
Course Title: Probability and Statistics
Course Instructors:RAD, BKP, SST Max. Time: 2 hours

Note: (a) All questions are compulsory.

(b) Marks are indicated against each question in square brackets.

(c) Define random variables along with range where applicable.

(d) Scientific calculators are allowed. Necessary statistical tables are supplied.

(e) The candidate is allowed to make Suitable numeric assumptions wherever required.

1. You go to see the doctor about an ingrowing toenail. The doctor selects you at random to have a blood test for *swine flu*, which is suspected to affect 1 in 10,000 people in a region. The test is 99% accurate, in the sense that the probability of a false positive is 1%. The probability of a false negative is zero. (5 Marks) [CO-1]

   (a) What is the probability that you test positive?

   (b) Given that you tested positive, what is the probability that you have swine flu?

2. Consider the continuous random variables $\mathbf{X}$ and $\mathbf{Y}$ with joint density: (5 Marks) [CO-3]

$$\mathtt{f}(x,\, y) \;=\; \begin{cases} 24xy & , \quad 0 < x < 1, 0 < y < 1, 0 < x+y < 1 \\ 0 & , \quad \textit{else} \end{cases}$$

   Assume that the marginal density function of $\mathbf{Y}$ is $\mathtt{f}_{\mathbf{Y}}(y) = 12y(1-y)^2$, $0 < y < 1$.

   (a) Find $\mathbb{P}(\mathbf{X} + \mathbf{Y} < 1/2)$.

   (b) Determine the conditional density of $\mathbf{X}$ given $\mathbf{Y} = \frac{1}{2}$.

3. In an industrial process, the diameter of a ball bearing is an important measurement. The buyer sets specifications for the diameter to be 3.0 ± 0.01 cm. The implication from the experience is that no part falling outside these specifications will be accepted. It is known that in the process the diameter of a ball bearing has a normal distribution with mean $\mu = 3.0$ and standard deviation $\sigma = 0.005$. On average, how many manufactured ball bearings will be scrapped? (5 Marks) [CO-3]

4. Find the best *least squares fit* to the data by a quadratic polynomial. (5 Marks) [CO-4]

x	-1	0	1	2
y	2	5	3	0

5. Consider the following dataset representing the programming skills (X) and corresponding exam scores (Y) for a group of B. Tech. computer science students. Assign ranks to the data and calculate the Spearman's Rank Correlation Coefficient and interpret your result. (5 Marks) [CO-4]

Student	Programming Skills (X)	Exam Scores (Y)
A	75	80
B	90	65
C	80	75
D	70	85

6. In comparing the variability of the tensile strength of two kinds of steel, an experiment yielded the following results: $n_1 = 6, s_1^2 = 19.2, n_2 = 4, s_2^2 = 3.5$, where the units of measurement are 1,000 pounds per square inch. Assuming that the measurements constitute independent random samples from two normal populations, test the null hypothesis $\sigma_1^2 = \sigma_2^2$ against the alternative $\sigma_1^2 \neq \sigma_2^2$ at the 0.05 level of significance. (5 Marks) [CO-5]

7. Use the data in the following table to test at the 0.01 level of significance whether a person's ability in mathematics is independent of his or her interest in statistics.: (5 Marks) [CO-5]

		Ability in mathematics		
		Low	*Average*	*High*
Interest in statistics	*Low*	63	42	15
	Average	58	61	31
	High	14	47	29

(Standard) Normal probability table to compute $\mathbb{P}(Z \leq z)$:

−2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
−2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
−2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183

F-distribution table:

	v_1								
v_2	1	2	3	4	5	6	7	8	9
1	161.45	199.50	215.71	224.58	230.16	233.99	236.77	238.88	240.54
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81

χ^2-distribution table:

	α									
v	0.30	0.25	0.20	0.10	0.05	0.025	0.02	0.01	0.005	0.001
1	1.074	1.323	1.642	2.706	3.841	5.024	5.412	6.635	7.879	10.827
2	2.408	2.773	3.219	4.605	5.991	7.378	7.824	9.210	10.597	13.815
3	3.665	4.108	4.642	6.251	7.815	9.348	9.837	11.345	12.838	16.266
4	4.878	5.385	5.989	7.779	9.488	11.143	11.668	13.277	14.860	18.466
5	6.064	6.626	7.289	9.236	11.070	12.832	13.388	15.086	16.750	20.515