JAYPEE UNIVERSITY OF INFORMATION TECHNOLOGY, WAKNAGHAT
TEST -2 EXAMINATION- APRIL-2023

COURSE CODE(CREDITS): 18B11MA413 (3)

COURSE NAME: DISCRETE MATHEMATICS

COURSE INSTRUCTOR: P K Pandey

MAX. MARKS: 25

MAX. TIME: 1 Hour 30 Minutes

Note: All questions are compulsory. Marks are indicated against each question in square brackets.

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1. (a) Obtain the generating function for the sequence $2, 0, 2, 0, 2, 0, \cdots$ [CO1] [2]

(b) Using the generating function, solve the following recurrence relation: [CO1] [3]

$$a_n = 2a_{n-1} + 1; n \geq 1, and\ a_0 = 1$$

2. (a) Using Euclidean algorithm find the gcd(7592, 5913). Moreover, answer whether $69x + 123y = 4$ has an integer solution or not? [CO4][3]

(b) Draw the Hasse Diagram of D_{100} under the partial order relation of divisibility. [2]

3. Using truth table, examine the validity of the following argument: [CO3][3]

If I try hard and I have talent, then I will become an engineer.
If I become an engineer, then I will be happy.

Therefore, if I will not be happy, then I did not try hard or I do not have talent.

4. Using mathematical induction show that $(ab)^n = a^n b^n, \forall n \in \mathbb{N}$. [CO1][3]

5. (a) Consider the predicate $P(x)$: $x \geq 2$; over the domain of real numbers. Determine the truth value of "$P(-4) \rightarrow P(2)$". Give reason in support of your answer. [CO2][1.5]

(b) Write the negation of $\forall x \forall y \exists z \ (x^2 + y^2 - 2z > 0)$. [CO2][1.5]

6. Verify whether $(p \wedge \sim q) \vee (p \vee q)$ is a tautology or not? [CO2][2]

7. Consider the poset $(D_{30}, |)$. For $a, b \in D_{30}$ define $a \vee b = lcm\ \{a, b\}$ and $a \wedge b = gcd\{a, b\}$. Now answer the following: [CO4][1+3]

(i) Draw the Hasse diagram of the poset $(D_{30}, |)$.

(ii) Obtain complement of each element in lattice $(D_{30}, \wedge, \vee)$. Give reasons whether this lattice is bounded, and complemented or not?