

Jaypee University of Information Technology, Waknaghat

Test-2 Examination - April 2024

B.Sc. (Mathematics and Computing) - II Semester

Course Code/Credits: 22BS1MA212/4

Max. Marks: 25

Course Title: Fundamentals of Probability and Statistics

Course Instructors: RAD

Max. Time: 90 minutes

Note: (a) ALL questions are compulsory.

(b) Scientific calculators are allowed to use.

(c) Marks are indicated against each question in round brackets.

(d) The candidate is allowed to make suitable numeric assumptions wherever required.

1. Consider frequency distribution of marks of 43 students of a university: (3 Marks) [CO-1]

Marks (x)	10	20	30	40	50	60
No. of Students (f)	4	7	15	8	7	2

(a) Find the third *quartile* Q_3 of the marks of students.

(b) Suppose $Q_1 = 20$. What is the *quartile deviation* of marks?

2. Out of those brought to court, there are 60% which are actually guilty. Of those that are guilty, 95% of them are convicted. But there are 1% of innocent people who get falsely convicted. (4 Marks) [CO-2]

(a) Determine the probability of you being convicted.

(b) What is the probability that you are actually innocent given that you are convicted?

3. Consider the density of the age of babies in years at a postnatal clinic: (4 Marks) [CO-3]

$$f(x) = \begin{cases} \frac{3}{4}x(2-x) & , 0 < x < 2 \\ 0 & , \text{else} \end{cases}$$

(a) What is the probability that a baby is under $2/3$ years old?

(b) Out of 60 babies brought, how many are expected to be under 8 months old?

4. Consider the *moment generating function* of a random variable X : (3 Marks) [CO-3]

$$M_X(t) = e^{t^2+3t}, \quad -\infty < t < \infty$$

Find the *mean* and *variance* of X .

5. Suppose that you attend an international conference with 500 delegates. (4 Marks) [CO-4]

(a) Without computing, write down the expression for the probability that exactly one other guest has the same birthday as you? For simplicity, exclude the possibility of a February 29 birthday.

(b) Approximate the probability expression obtained by using a Poisson distribution.

6. Consider a job-arrival computer system with $\lambda = 2$ arrivals per minute. (4 Marks) [CO-4]
- (a) Find the probability of at most 3 arrivals in any one-minute interval.
 - (b) What is the maximum jobs that should arrive in one minute with 90% certainty?
7. Let $\mathbf{X} \sim \text{Gamma}(\alpha, \beta)$ be a random variable with $\mu = 20$ and $\sigma^2 = 100$. (3 Marks) [CO-4]
- (a) Determine the parameters α and β .
 - (b) Describe the random variable \mathbf{X} and write down its *probability density function* $f(x)$.

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