

Jaypee University of Information Technology, Waknaghat

Test-2 Examination, April 2024

B.Tech - II Semester (CSE/CSE-AIML/CSE-AIDS/CSE-CS/IT/ECE/CE)

Course Code/Credits: 18B11MA211/4

Course Title: Engineering Mathematics - II

Course Instructors: RAD*, BKP, PKP, MDS, SST

Max. Marks: 25

Max. Time: 90 min

Note: (a) All questions are compulsory.

(b) Scientific calculators are allowed.

(c) Marks are indicated against each question in round brackets.

(d) The candidate is allowed to make suitable numeric assumptions wherever required.

1. Consider the following non-positive term series:

(3 Marks) [CO-1]

$$\sum_{n=2}^{\infty} a_n = \frac{1}{\log 2} - \frac{1}{\log 3} + \frac{1}{\log 4} - \dots$$

Apply Ratio test on the positive-term series $\sum |a_n|$ to see if the given $\sum a_n$ converges.

2. Consider the Bessel function of order 0:

(2 Marks) [CO-1]

$$J_0(x) := \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(n!)^2 2^{2n}}$$

Determine the *radius of convergence* of $J_0(x)$.

3. Consider the constant function $f(x) = 1$ defined on $(0, \pi)$.

(3 Marks) [CO-1]

(a) Show that $f(x) = \frac{4}{\pi} \left[\sin x + \frac{1}{3} \sin 3x + \frac{1}{5} \sin 5x + \dots \right]$ is the half-range sine series.

(b) Deduce $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots$ by Parseval's identity.

4. Consider the following constant-coefficient differential equation:

(3 Marks) [CO-2]

$$D(D-1)^2(D^2+1)y = 0$$

(a) Determine the *order* of the differential equation.

(b) Write down its general solution.

5. Consider the following Cauchy-Euler differential equation:

(3 Marks) [CO-2]

$$x^3 \frac{d^3 y}{dx^3} + 6x^2 \frac{d^2 y}{dx^2} - 12y = \frac{12}{x^2}$$

(a) Convert the given differential equation into constant-coefficient differential equation.

(b) Determine the *complementary solution* of the resulting differential equation.

6. Consider $(D^2 - 3D + 2)y = e^{3x}$. Let $y_h(x)$ be the *homogeneous solution*. (4 Marks) [CO-2]

(a) Let $y_h(x) = c_1 e^x + c_2 e^{-3x}$. Find the particular integral y_p using *variation of parameter*.

(b) Compute $y(x)$ satisfying $y(0) = 1$ and $y'(0) = 0$.

7. Consider the following differential equation:

(4 Marks) [CO-3]

$$\frac{d^2y}{dx^2} - 3x^2 \frac{dy}{dx} = 0$$

- (a) Find the ordinary points of the differential equation.
- (b) Obtain the series solution about the point $x = 0$.

8. Let $\mathcal{P}_n(x)$ be the Legendre polynomial of degree n .

(3 Marks) [CO-3]

- (a) Write down the Legendre polynomials $\mathcal{P}_n(x)$ for $n = 0, 1, 2, 3$.
- (b) Express $\mathcal{P}(x) = 3\mathcal{P}_3(x) + 2\mathcal{P}_2(x) + 4\mathcal{P}_1(x) + 5\mathcal{P}_0(x)$ as a polynomial in x .

* * * * *

JUT TEST-2 EXAMINATIONS - APRIL 2024