Jaypee University of Information Technology, Waknaghat

Test-3 Examination, May 2024

B.Tech - II Semester (CSE/CSE-AIML/CSE-AIDS/CSE-CS/IT/ECE/CE)

Course Code/Credits: 18B11MA211/4
Course Title: Engineering Mathematics - II
Course Instructors: RAD*, BKP, PKP, MDS, SST

Max. Marks: 35
Max. Time: 2 hours

Note: (a) All questions are compulsory.

(b) Scientific calculators are allowed.

(c) Marks are indicated against each question in round brackets.

(d) The candidate is allowed to make suitable numeric assumptions wherever required.

1. Test the series for convergence: $\frac{\sqrt{2}-\sqrt{1}}{1} - \frac{\sqrt{3}-\sqrt{2}}{2} + \frac{\sqrt{4}-\sqrt{3}}{3} - \frac{\sqrt{5}-\sqrt{4}}{4} + \cdots$. (4 Marks) [CO-1]

2. Consider the differential equation: $y'' - xy' + 3y = 0$. (4 Marks) [CO-3]

   (a) Obtain a power series solution about the point $x = 0$.

   (b) Explain whether the point $x = 0$ is an ordinary point or a regular singular point.

3. The vibration of an elastic string is governed by the following PDE: (5 Marks) [CO-4]

$$\frac{\partial^2 \mathrm{u}}{\partial \mathrm{t}^2} = \frac{\partial^2 \mathrm{u}}{\partial x^2}$$

   The length of the string is π and the ends are fixed. The initial velocity is zero and the initial deflection is $\mathrm{u}(x,\ 0) = 2(\sin x + \sin 3x)$. Find deflection $\mathrm{u}(x,t)$ of the vibrating string for $\mathrm{t} > 0$.

4. Show that the function $f(z)$ is not continuous at $z = 0$, where (4 Marks) [CO-5]

$$f(z) = \begin{cases} \frac{\mathrm{Re}(z^2)}{|z|^2} & , \ z \neq 0 \\ z & , \ z = 0 \end{cases}$$

5. Show that $f(z)$ is not analytic at $z = 0$ although C-R equations are satisfied at the origin:

$$f(z) = \begin{cases} \frac{x^3(y-ix)}{x^6+y^2} & , \ z \neq 0 \\ 0 & , \ z = 0 \end{cases}$$

   (4 Marks) [CO-5]

6. Evaluate the integral $\oint_C \frac{e^{2z}}{(z+1)^2}$ over $C : |z-1| = \frac{7}{2}$. (4 Marks) [CO-6]

7. Consider $f(z) = \frac{1}{(z+1)(z^2+2)}$. (5 Marks) [CO-6]

   (a) Identify the singular points. Classify the singular point $z = \mathrm{i}\sqrt{2}$.

   (b) Expand $f(z)$ in Laurent's series valid for $1 < |z| < \sqrt{2}$.

8. Consider $\int_0^{2\pi} \frac{1}{2+\sin\theta}\, d\theta$. (5 Marks) [CO-7]

   (a) Transform the given real integral into a complex integral over the unit-circle.

   (b) Evaluate the transformed complex integral using Cauchy's residue theorem.

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