CARRIER FREQUENCY OFFSET ANALYSIS OF OFDM SYSTEMS

Project Report submitted in partial fulfillment of the requirement
for the degree of

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in

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under the supervision of

Mr. Bhasker Gupta

By

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CERTIFICATE

This is to certify that project report entitled **"CARRIER FREQUENCY OFFSET ANALYSIS OF OFDM SYSTEMS"**, submitted by **Hitesh Sarup (091025), Arnav Ghildial (091087) & Isha Tuli (091088)** in partial fulfillment for the award of degree of Bachelor of Technology in Electronics and Communication Engineering to Jaypee University of Information Technology, Waknaghat, Solan has been carried out under my supervision.

This work has not been submitted partially or fully to any other University or Institute for the award of this or any other degree or diploma.

Date: 30/5/2013

Supervisor's Name: Mr. Bhasker Gupta

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ACKNOWLEDGEMENT

We **Hitesh Sarup, Arnav Ghildial and Isha Tuli** pursuing B.Tech in Electronics and Communication Engineering from Jaypee University Of Information Technology, Waknaghat, Distt. Solan, Himachal Pradesh; would like to give a vote of thanks to **Mr. Bhasker Gupta** who gave us the right guidance and opportunity to do a project on "**CARRIER FREQUENCY OFFSET ANALYSIS OF OFDM SYSTEMS**" under him. This project will help us to understand practically as well as conceptually the various applications of orthogonal carriers and frequency division multiplexing which will help us in our B.Tech degree as well as future endeavors. Therefore we are highly obliged by the valuable knowledge we are getting from him and the university and we would like to implement it in our future field related works.

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ABSTRACT

This project discusses and investigates the estimation of carrier frequency offset inorthogonal frequency division multiplexing (OFDM). **We have discussed about the various aspects of OFDM transmitter and receiver structure, studied the OFDM signal response through AWGN channel** in theory as well as in code implementation which will help us to understand carrier frequency offset estimation even better.

Although OFDM is resistant to multipath fading, it requires a high degree of synchronization to maintain sub-carrier orthogonality. Therefore the level of performance of the system depends on the accuracy in estimating the carrier frequency offset.Redundant information contained within the cyclic prefix enables this estimation without additional pilots. So **we have also corrected our OFDM signal with the help of two carrier frequency offset estimation techniques (using preamble and cyclic prefix) and then compared the BER of the signals passed through them and the two originals.**

The purpose of this report is to provide the following information concerning OFDM:

- Theory of operation
- Analysis of important characteristics
- Implementation using MATLAB

CHAPTER 1

Introduction

1.1 Purpose:-

Efficient use of radio spectrum includes placing modulated carriers as close as possible without causing Inter-Carrier Interference (ICI). Optimally, the bandwidth of each carrier would be adjacent to its neighbors, so there would be no wasted spectrum. In practice though, a guard band must be placed between each carrier bandwidth to provide a space where a filter can attenuate an adjacent carrier's signal. These guard bands resulted in wasted bandwidth.

In order to transmit high data rates, short symbol periods must be used. The symbol period is the inverse of the baseband data rate ($T = 1/R$), so as R increases, T must decrease. In a multi-path environment, a shorter symbol period leads to a greater chance for Inter-Symbol Interference (ISI). This occurs when a delayed version of symbol 'n' arrives during the processing period of symbol 'n+1'.

Orthogonal Frequency Division Multiplexing (OFDM) addresses both of these problems. OFDM provides a technique allowing the bandwidths of modulated carriers to overlap without interference (no ICI). It also provides a high date rate with long symbol duration, thus helping to eliminate ISI.

1.2 OFDM Overview:-

OFDM is a modulation technique where multiple low data rate carriers are combined by a transmitter to form a composite high data rate transmission. Digital signal processing makes OFDM possible. To implement the multiple carrier scheme using a bank of parallel modulators would not be very efficient in analog hardware. However, in the digital domain, multi-carrier modulation can be done efficiently with currently available digital signal processing hardware and software. Not only can it be done, but it can also be made very flexible and programmable. This allows OFDM to make maximum use of available bandwidth and to be able to adapt to changing system requirements. Each carrier in an OFDM system is a sinusoid with a frequency that is an integer multiple of a base or fundamental sinusoid frequency.

Therefore, each carrier is like a Fourier series component of the composite signal. In fact, it will be shown later that an OFDM signal is created in the frequency domain, and then transformed into the time domain via the Discrete Fourier Transform (DFT).

Two periodic signals are orthogonal when the integral of their product, over one period, is equal to zero. This is true of certain sinusoids as illustrated in equation1.

Continuous Time :

$$\int_0^T \cos(2\pi n f_0 t) \times \cos(2\pi m f_0 t) dt = 0 \quad (\mathrm{n} \neq \mathrm{m})$$

Discrete Time :

$$\sum_{\mathrm{k}=0}^{\mathrm{N-1}} \cos\left(\frac{2\pi \mathrm{kn}}{\mathrm{N}}\right) \times \cos\left(\frac{2\pi \mathrm{km}}{\mathrm{N}}\right) = 0 \quad (\mathrm{n} \neq \mathrm{m})$$

----------- (1)

Where - t= continuous time constant,

k=integer time constant

The carriers of an OFDM system are sinusoids that meet this requirement because each one is a multiple of a fundamental frequency. Each one has an integer number of cycles in the fundamental period.

1.3 OFDM Modulation:-

It is mapping of the information on changes in the carrier phase, frequency or amplitude or a combination. Binary data from a memory device or from a digital processing stream is used as the modulating (baseband) signal. The following steps may be carried out in order to apply modulation to the carriers in OFDM:

- Combine the binary data into symbols according to the number of bits/symbol selected

- Convert the serial symbol stream into parallel segments according to the number of carriers, and form carrier symbol sequences
- Convert each symbol into a complex phase representation
- Take the IFFT of the result

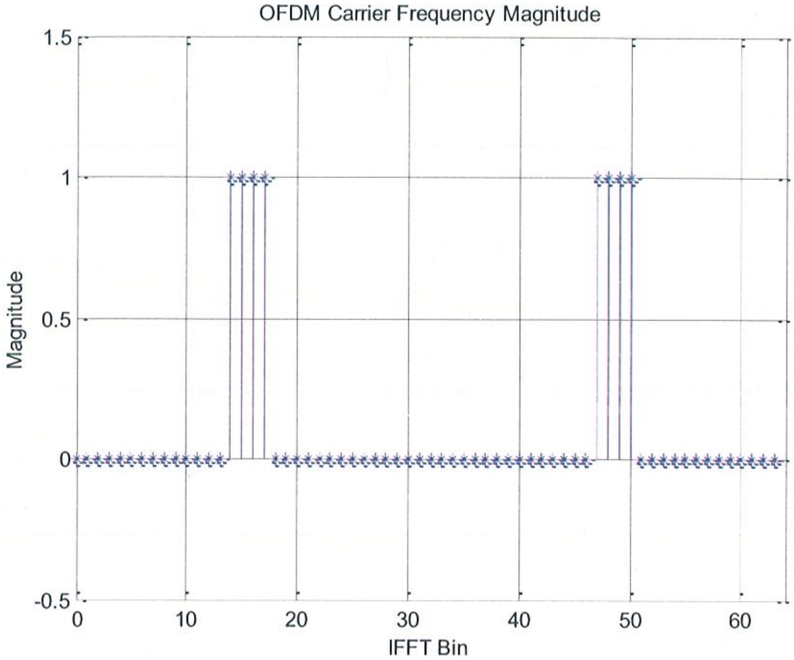

Fig1: OFDM Carrier Magnitude prior to IFFT

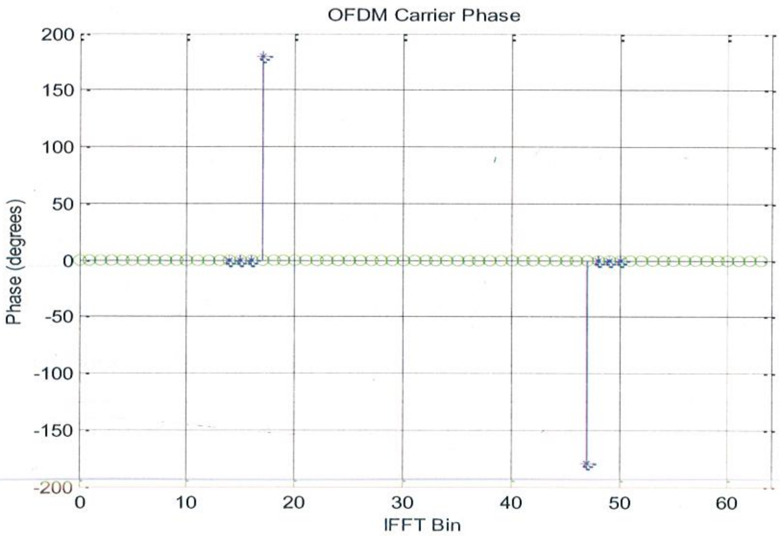

Fig2: OFDM Carrier Phase prior to IFFT

Note that the modulated OFDM signal is nothing more than a group of delta (impulse) functions, each with a phase determined by the modulating symbol. In addition, note that the frequency separation between each delta is proportional to 1/N where N is the number of IFFT bins. The frequency domain representation of the OFDM is described in equation 2

$$S(k) = e^{j\theta_m}\delta\left(k - m - \frac{N}{2}\right) + e^{-j\theta_m}\delta\left(k + m - \frac{N}{2}\right) \quad \text{single (real) OFDM modulated carrier}$$

$$\begin{aligned} k &= \text{frequency (0 to N - 1)} \\ m &= \text{OFDM carrier frequency} \\ N &= \text{IFFT bin size} \end{aligned}$$

$$S(k)_{ofdm} = \sum_{m=c_{first}}^{c_{last}} \left[e^{j\theta_m}\delta\left(k - m - \frac{N}{2}\right) + e^{-j\theta_m}\delta\left(k + m - \frac{N}{2}\right) \right] \quad \text{composite OFDM modulated carriers}$$

$$c = \text{OFDM carrier}$$

---------- (2)

After the modulation is applied, an IFFT is performed to generate one symbol period in the time domain. It is clear that the OFDM signal has varying amplitude. It is very important that the amplitude variations be kept intact as they define the content of the signal. If the amplitude is clipped or modified, then an FFT of the signal would no longer result in the original frequency characteristics, and the modulation may be lost.

The time domain representation of the OFDM signal is given in as-

$$s(n) = \sum_{m=c_{first}}^{c_{last}} \sum_{n=0}^{N-1} \cos\left(\frac{2\pi mn}{N} + \theta_m\right)$$

n = time sample

m = OFDM carrier

N = IFFT bin size

θ_m = phase modulation for OFDM carrier (m)

c_{first}, c_{last} = OFDM carriers (first and last)

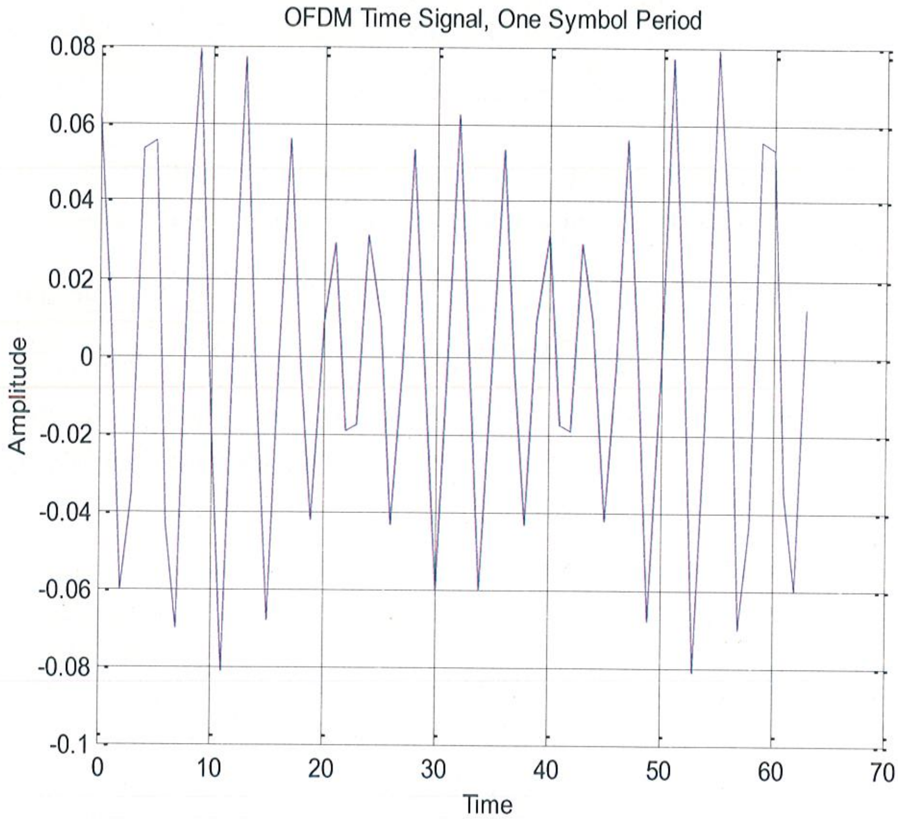

Fig 3: OFDM signal in the form of sinusoids

1.4 OFDM operation:-

1.4.1 Preliminary Concepts:

When the DFT (Discrete Fourier Transform) of a time signal is taken, the frequency domain results are a function of the time sampling period and the number of samples and an example is shown in Fig 4. The fundamental frequency of the DFT is equal to 1/NT (1/total sample time). Each frequency represented in the DFT is an integer multiple of the fundamental frequency. The maximum frequency that can be represented by a time signal sampled at rate 1/T is f_{max} = ½T as given by the Nyquist sampling theorem. This frequency is located in the center of the DFT points. All frequencies beyond that point are images of the representative frequencies.

The IDFT (Inverse Discrete Fourier Transform) performs the opposite operation to the DFT. It takes a signal defined by frequency components and converts them to a time signal. The parameter mapping is the same as for the DFT. The time duration of the IDFT time signal is equal to the number of DFT bins (N) times the sampling period (T).

It is perfectly valid to generate a signal in the frequency domain, and convert it to a time domain equivalent for practical use*. This is how modulation is applied in OFDM.

> * The frequency domain is a mathematical tool used for analysis. Anything usable by the real world must be converted into a real, time domain signal.

In practice the Fast Fourier Transform (FFT) and IFFT are used in place of the DFT and IDFT, so all further references will be to FFT and IFFT.

1.4.2Transmission:

The key to the uniqueness and desirability of OFDM is the relationship between the carrier frequencies and the symbol rate. Each carrier frequency is separated by a multiple of 1/NT (Hz). The symbol rate for each carrier is 1/NT (symbols/sec).

The effect of the symbol rate on each OFDM carrier is to add a sin(x)/x shape to each carrier's spectrum. The nulls of the sin(x)/x (for each carrier) are at integer multiples of 1/NT. The peak (for each carrier) is at the carrier frequency k/NT. Therefore, each carrier frequency is located at the nulls for all the other carriers. This means that none of the carriers will interfere with each other during transmission, although their spectrums overlap. The ability to space carriers so closely together is very bandwidth efficient.

Fig. 5 shows the spectrum for of an OFDM signal with the following characteristics:

- 1 bit / symbol
- 100 symbols / carrier (i.e. a sequence of 100 symbol periods)
- 4 carriers
- 64 IFFT bins
- spectrum averaged for every 20 symbols (100/20 = 5 averages)

Note that the nulls of the spectrums line up with the unused frequencies. The four active carriers each have peaks at carrier frequencies. It is clear that the active carriers have nulls in their spectrums at each of the unused frequencies (otherwise, the nulls would not exist). Although it cannot be seen in the figure, the active frequencies also have spectral nulls at the adjacent active frequencies. Fig 4 shows the OFDM time waveform for the same signal. There are 100 symbol periods in the signal. Each symbol period is 64 samples long (100 x 64 = 6400 total samples), carriers each of which carries 1 symbol and carries 1 bit. Note that Fig. 5 again illustrates the large dynamic range of the OFDM waveform envelope.

It is not currently practical to generate the OFDM signal directly at RF rates, so it must be up converted for transmission. To remain in the discrete domain, the OFDM could be up sampled and added to a discrete carrier frequency. This carrier could be an intermediate frequency whose sample rate is handled by current technology. It could then be converted to analog and increased to the final transmit frequency using analog frequency conversion methods. Alternatively, the OFDM modulation could be immediately converted to analog and directly increased to the desired RF transmit frequency.

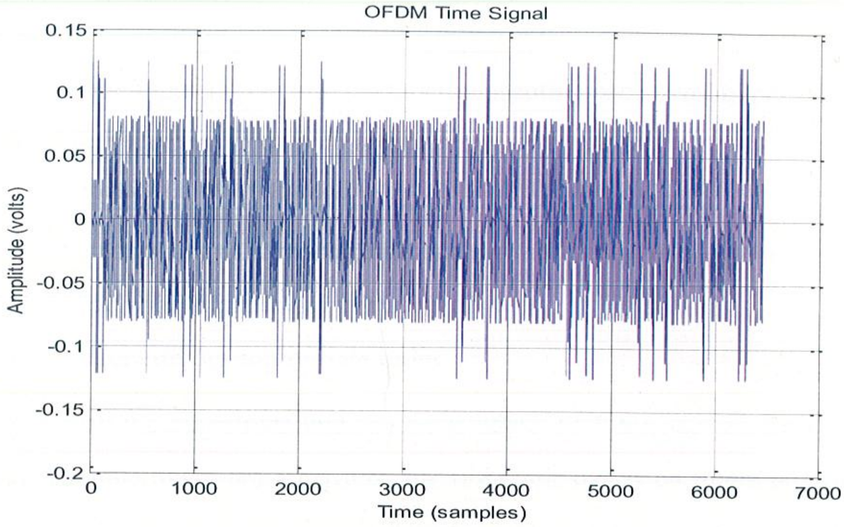

Fig4: OFDM Time Waveform

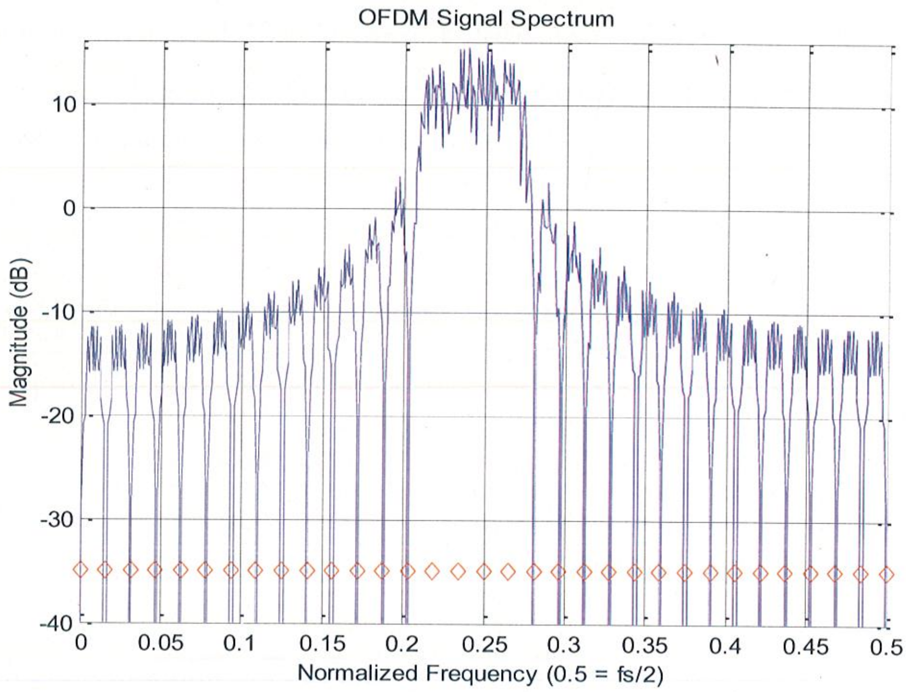

Fig5: OFDM Spectrum

1.4.3 Reception and Demodulation:

The received OFDM signal is down converted (in frequency) and taken from analog to digital. Demodulation is done in the frequency domain (just as modulation was). The following steps may be taken to demodulate the OFDM:

- Partition the input stream into vectors representing each symbol period
- Take the FFT of each symbol period vector
- Extract the carrier FFT bins and calculate the phase of each
- Calculate the phase difference, from one symbol period to the next, for each carrier
- Decode each phase into binary data
- Sort the data into the appropriate order

Fig. 6 and Fig. 7show the magnitude and spectrum of the FFT for one received OFDM symbol period. For this example, there are 4 carriers, the IFFT bin size is 64, there is 1 bit per symbol, and the signal was sent through a AWGN channel having an SNR of 8 dB. The figures show that, under these conditions, the modulated symbols are very easy to recover. Note in Fig.7that the unused frequency bins contain widely varying phase values.

Even if the noise is removed from the channel, these phase variations still occur. It must be a result of the IFFT/FFT operations generating very small complex values (very close to 0) for the unused carriers. The phases are a result of these values.

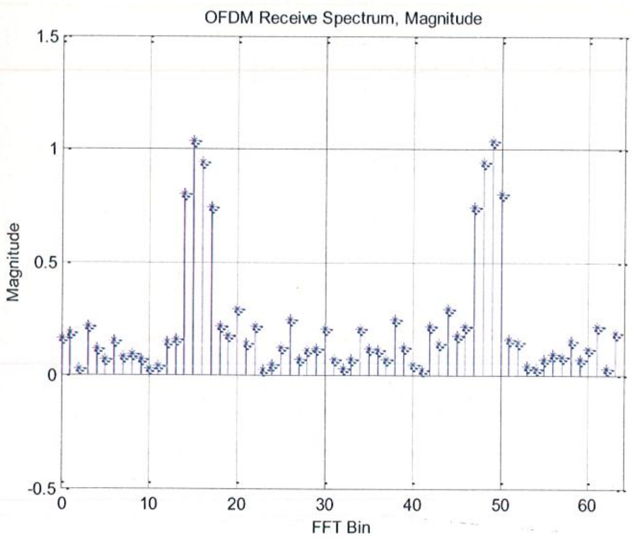

Fig 6: OFDM Carrier Magnitude following FFT

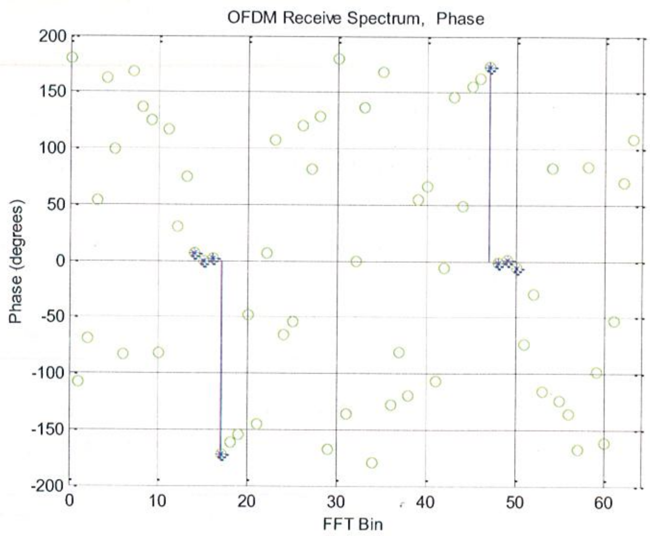

Fig 7: OFDM Carrier Phase following FFT

1.4.4 Guard Period:

OFDM demodulation must be synchronized with the start and end of the transmitted symbol period. If it is not, then ISI will occur (since information will be decoded and combined for 2 adjacent symbol periods). ICI will also occur because orthogonality will be lost (integrals of the carrier products will no longer be zero over the integration period).

To solve this problem, a guard interval is added to each OFDM symbol period. The first method is to simply make the symbol period longer, so that the demodulator does not have to be so precise in picking the period beginning and period end, and decoding is always done inside a single period. This would fix the ISI problem, but not the ICI problem. If a complete period is not integrated (via FFT), orthogonality will be lost. In order to avoid ISI and ICI, the guard period must be formed by a cyclic extension of the symbol period. This is done by taking symbol period samples from the end of the period and appending them to the front of the period. The concept of being able to do this, and what it means, comes from the nature of the IFFT/FFT process. When the IFFT is taken for a symbol period (during OFDM modulation), the resulting time sample sequence is technically periodic. This is because the IFFT/FFT is an extension of the Fourier Transform which is an extension of the Fourier series for periodic waveforms. All of these transforms operate on signals with either real or manufactured periodicity. For the IFFT/FFT, the period is the number of samples used. With the cyclic extension, the symbol period is longer, but it represents the exact same frequency spectrum. As long as the correct number of samples are taken for the decoding, they may be taken anywhere within the extended symbol. Since a complete period is integrated, orthogonality is maintained. Therefore, both ISI and ICI are eliminated.

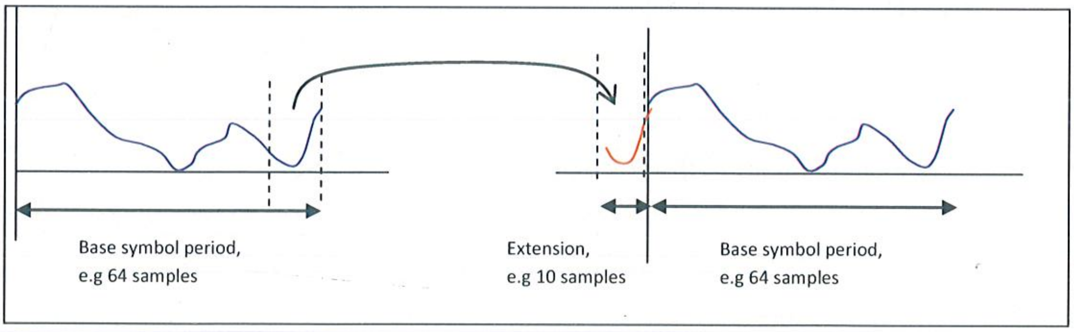

Fig 8: Guard Period via Cyclic Extension

CHAPTER 2

Orthogonal Frequency Division Multiplexing (OFDM)

Multiplexing is a method of sharing bandwidth with many independent data channels. Multiplexing generally refers to independent signals, those produced by different sources. OFDM is a combination of modulation and multiplexing. In OFDM the concept of multiplexing is applied to independent signals but these signals are a subset of the one main signal. In OFDM the signal itself is split into independent channels, modulated by data and these re-multiplexed to create the OFDM carrier. OFDM is a special case of Frequency Division Multiplex (FDM).

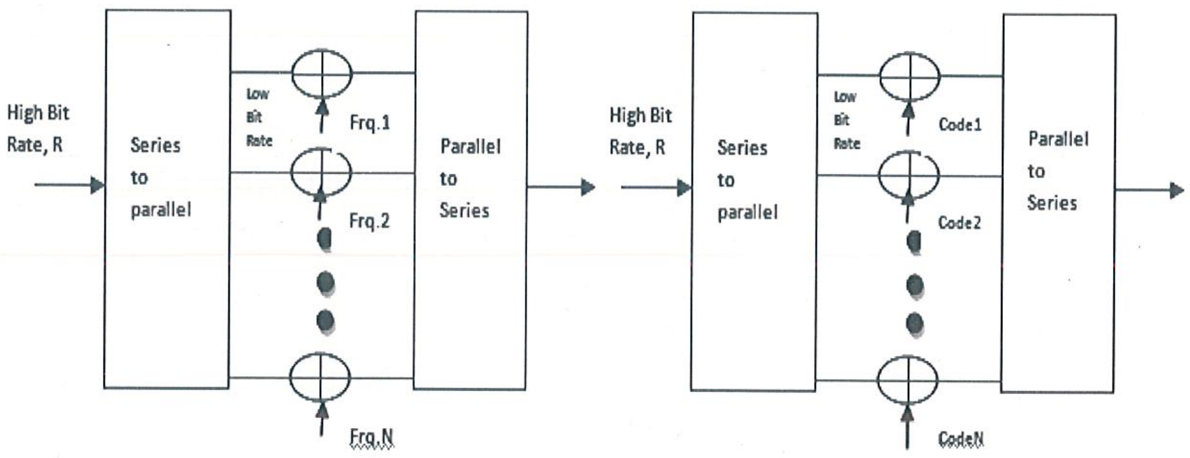

Fig 9: Multicarrier FDM and multi code division multiplexing

2.1 The importance of being orthogonal:-

The main concept in OFDM is orthogonality of the sub-carriers. Since the carriers are all sine/cosine wave, area under one period of a sine or a cosine wave is zero.

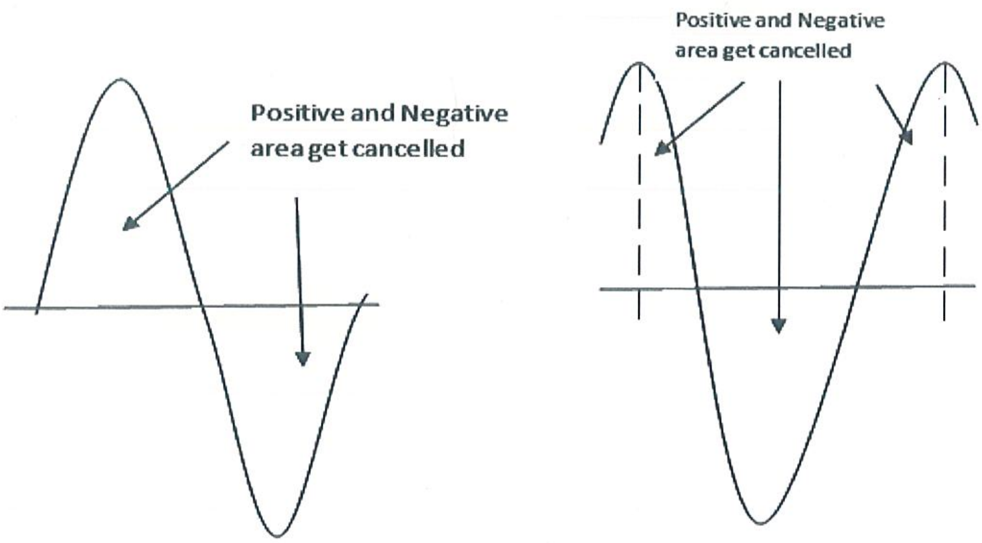

Fig 10: The area under a sine and a cosine wave over one period is always zero

The orthogonality allows simultaneous transmission on a lot of sub-carriers in a tight frequency space without interference from each other. In essence this is similar to CDMA, where codes are used to make data sequences independent (also orthogonal) which allows many independent users to transmit in same space successfully.

2.2 OFDM a special case of FDM:-

If we have a bandwidth that goes from frequency a to b, and it is sub divided into a frequency space of four equal spaces.

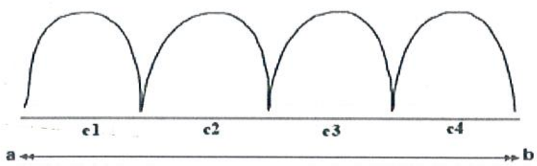

Fig. 11: FDM carriers are placed to next to each other

The frequencies a and b can be anything, integer or non-integer since no relations is implied between a and b. Same is true of the carrier center frequencies which are based on frequencies that do not have any special relationship to each other.

But what if frequency c_1 and c_n were such that for any integer n the following holds.

$$\mathbf{c_n = n \times c_1}$$

So that,

$$c_2 = 2c_1$$

$$c_3 = 3c_1$$

$$c_4 = 4c_1$$

All three of these frequencies are harmonic to c1. In this case, since these carriers are orthogonal to each other, when added together, they do not interfere with each other. In FDM, since we do not generally have frequencies that follow the above relationship, we get interference from neighbor carriers. To provide adjacent channel interference protection, signals are moved further apart.

2.3 Use of inverse FFT to create the OFDM symbol:-

Forward FFT takes a random signal, multiplies it successfully by complex exponentials over the range of frequencies, sums each product and plots the results as a coefficient of that frequency. The coefficients are called a spectrum and represent "how much" of that frequency is present in the input signal. The results of the FFT in common understanding is a frequency domain signal.

We can write FFT in sinusoids as-

$$x(k) = \sum_{n=0}^{N-1} x(n) \sin\left(\frac{2\pi kn}{N}\right) + j \sum_{n=0}^{N-1} x(n) \cos\left(\frac{2\pi kn}{N}\right)$$

Here x(n) are the coefficients of the sines and cosines of frequency 2лk/N, where k is index of the frequencies over the N frequencies, and n is the time index. X(k) is the value of the spectrum for the kth frequency and x(n) is the value of the signal at time n.

The inverse FFT takes this spectrum and converts the whole thing back to time domain signal by again successfully multiplying it by a range of sinusoids.

The equation for IFFT is-

$$X(n) = \sum_{n=0}^{N-1} x(k)\sin(\frac{2\pi kn}{N}) - j\sum_{n=0}^{N-1} x(k)\cos(\frac{2\pi kn}{N})$$

The two processes are a linear pair. Using both in sequence will give the original result back.

2.4 Delay spreads:-

Delay spread is like the undesired splashes might get from the car ahead of us. In composite, these splashes become noise and affect the beginning of the next symbol. To mitigate this noise at the front of the symbol, the symbol is moved further away from the region of delay spread as shown below-

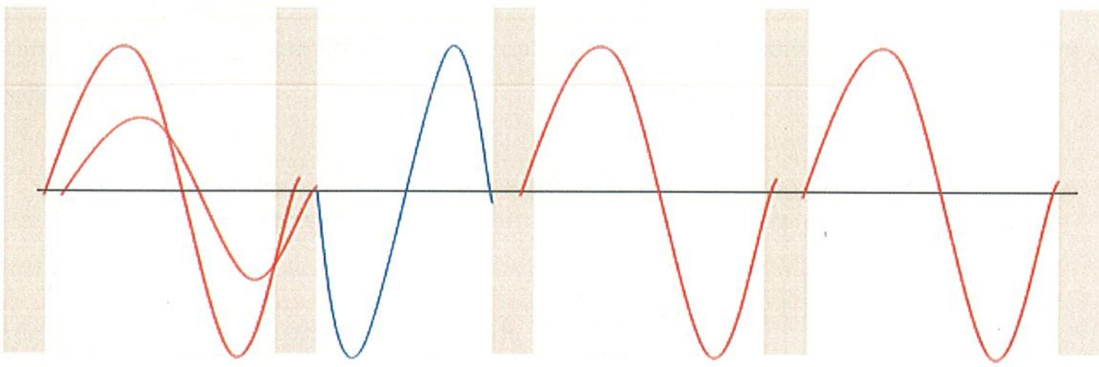

Fig 12: moving the symbol back so the arriving delayed signal peters out. So no interference in the next symbol

2.5 Adding a cyclic prefix:-

Cyclic prefix is this superfluous bit of signal we add to the front of the previous symbol. Since OFDM, has a lot of carriers, we would do this to each and every carrier. But in reality since the OFDM signal is a linear combination, we can add cyclic prefix just once to the composite OFDM signal. The prefix is taken as 25% of the symbol time.

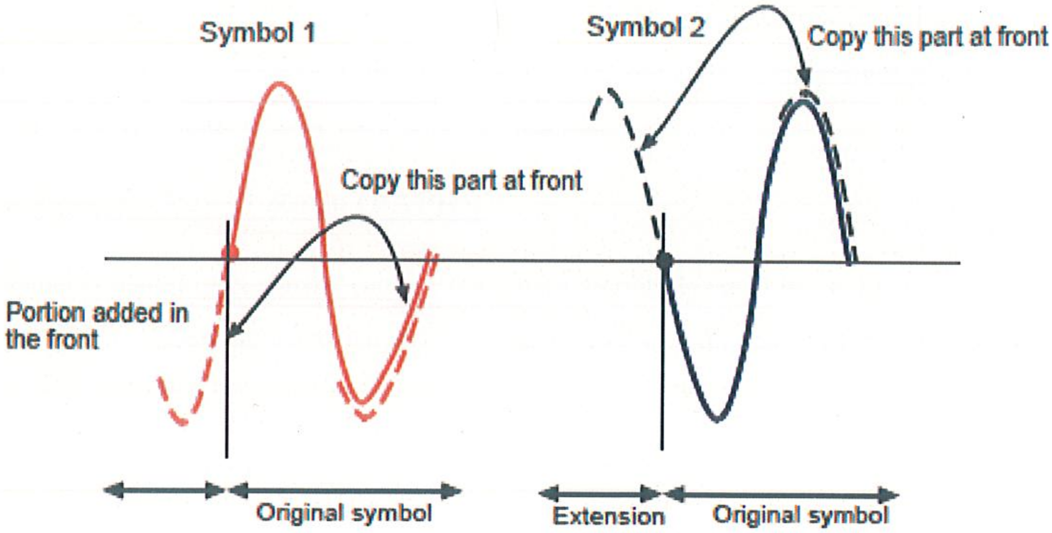

Fig13: cyclic prefix

Adding cyclic prefix to the OFDM signal further improves its ability to deal with fading and interference.

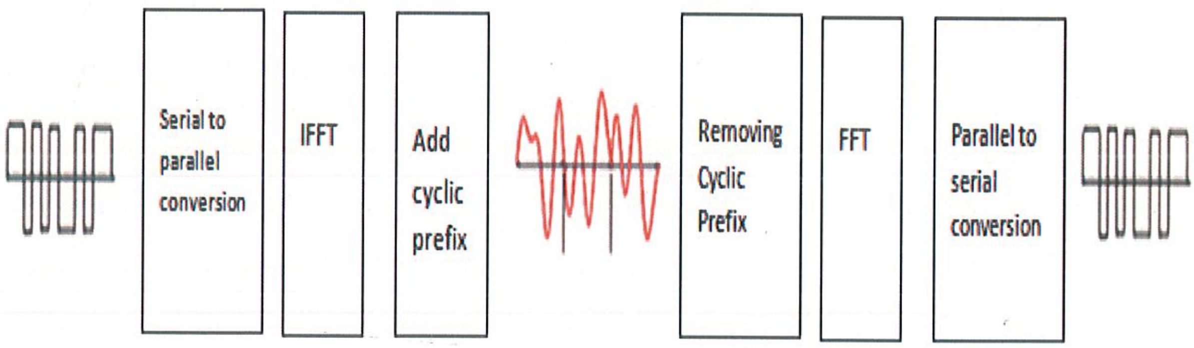

Fig 14: adding of cyclic prefix to modulated/demodulated signal

2.6 Properties of OFDM systems:-

2.6.1 BER (bit error rate) performance:

The BER of an OFDM is only exemplary in a fading environment. OFDM signal due to its amplitude variation does not behave well in a non-linear channel such as created by high power amplifiers on board satellites. For example using OFDM for a satellite would require a fairly large back off, on the order of 3 dB, so there must be some other compelling reason for its use such as when the signal is to be used for a moving user.

2.6.2 Peak To Power Ratio (PAPR):

If a signal is a sum of N signals each of maximum amplitude equal to 1v, then it is conceivable that maximum amplitude of N that is all N signals add at a moment at their maximum points. The PAPR is defined as-

$$R = \frac{|x(t)|^2}{P_{avg}}$$

For an OFDM signal that has 128 carriers, each with a normalized power of 1watts, then the max. PAPR can be as large as log(128) or 21 dB. This is at the instant when all 128 carriers combine at their max. points. The RMS PAPR will be around half of this number or 10-15 dB.

The other problem is that tight synchronization is needed. Often pilot tones are served in the sub-carrier space. These are used to lock on phase and to equalize the channel.

2.6.3 Modern OFDM:

The OFDM use has increased greatly in the last 10 years. It is now proposed for radio broadcasting such as Eureka 147 standard and Digital Radio Mondiale (DRM). OFDM is used for modem/ ADSL application where it coexists with phone line. For ADSL use, the channel, the phone line, is filtered to provide a high SNR. OFDM here is called Discrete Multi Tone (DMT). OFDM is also used in wireless internet modem and this usage is called 801.11a.

2.7 The OFDM system:-

The details of OFDM transmitter and receiver structure are shown in the block diagram below. OFDM systems basically involve transmission of a cyclic prefixed signal over a fading multipath channel.

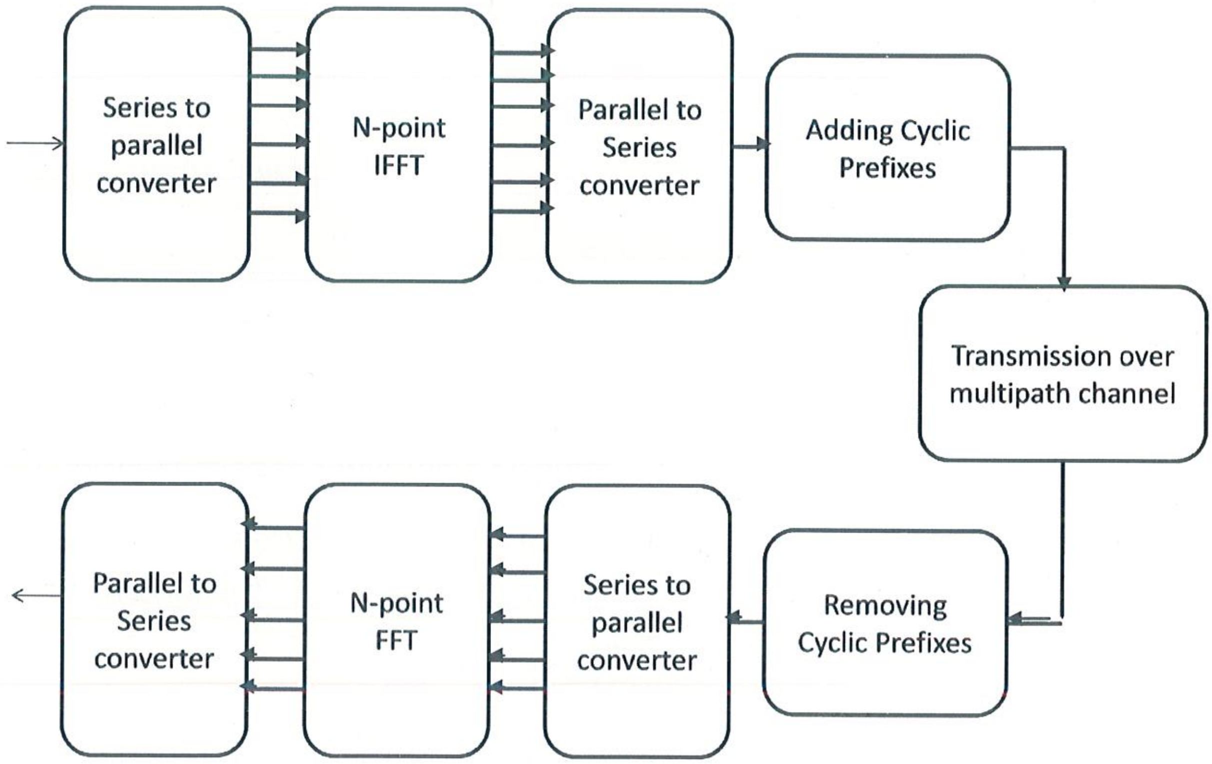

Fig 15: OFDM transmitter and receiver structure

2.7.1 The initial block:

The input symbols $\{s_k(i)\}_{i=1}^{N}$ denotes the transmit symbols for the k^th^ OFDM block. These symbols may come for instance from a M-QAM constellation or any random matrix.

2.7.2 N-ptIFFT block:

N denotes the number of OFDM sub-carriers (the number of constellation symbols to be transmitted in one OFDM block). After serial to parallel conversion of the input symbol stream, a N-pt IFFT is taken to get-

$$\{x_k(i)\}_{i=1}^{N}$$

2.7.3 Cyclic prefix block:

After parallel to serial conversion, a cyclic redundancy of length v (the number of CP samples) is added as a prefix in such a way that $x_k(-i) = x_k(N-i)$ for $i=1,2, \ldots\ldots, v$.

2.7.4 Transmission over a multipath channel:

The signal is then transmitted on a multipath channel with the Channel Impulse Response (CIR) of the multipath channel L denoted here by the vector-

$$h = [h_0 \quad h_1 \quad \ldots\ldots \quad h_{L-1}]^T \in C^L$$

The typical convolution equation for the k^{th} channel output symbol is-

$$y_k = [y_k(0) \quad y_k(1) \quad \ldots \quad y_k(N-1)]^T \in C^N$$

This can be expressed in matrix notation in terms of transmitted samples and noise vector

$$\tilde{\eta}_k \in C^N$$

$$\begin{bmatrix} y_k(0) \\ y_k(1) \\ \vdots \\ \vdots \\ y_k(N-1) \end{bmatrix} = \begin{bmatrix} h_{L-1} & \cdots & h_0 & \cdots & 0 \\ 0 & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & \cdots & h_{L-1} & \cdots & h_0 \end{bmatrix} \begin{bmatrix} x_{k-1}(N-E) \\ \vdots \\ x_{k-1}(N-1) \\ x_k(-v) \\ \vdots \\ x_k(0) \\ \vdots \\ \vdots \\ x_k(N-1) \end{bmatrix} + \overline{\eta}_k$$

Where E = L – v – 1 is the channel length exceeding the duration of cyclic prefix v. If CIR length L exceeds the duration of CP, i.e. E>0 the entities generated contribute to what is called the Inter Symbol Interference (ISI).

2.7.5 Receiver Section:

For the simplifications, consider the CP length to be greater than CIR length and forget about the insufficient CP scenario. The effective N x N channel matrix now gets circulant i.e. its rows are circularly shifted version of each other. This results in major simplifications after the receiver takes the FFT after CP removal. However, this circulant nature of the effective channel matrix is void if the channel is time variant, because in that case the CIR coefficients appearing in a row (corresponding to a sample of the OFDM symbol) are potentially different than the CIR coefficients appearing in some other row.

The matrix H is defined to be a diagonal matrix containing the Channel Frequency Response (CFR) coefficients along its main diagonal. We see that the final system model equation will reduce to –

$$Y_k = HX_k + \eta_k$$

The final multipath channel boils down to a number of interference-free parallel sub-channels whereby, each of the received sub-carrier can be given as the corresponding transmitted sub-carrier scaled by a scaler complex fading coefficient (CFR at the sub-carrier) and corrupted by the additive noise. The detection scheme at the receiver can be as simple as just dividing the received sub-carrier by the estimated Channel Frequency Response.

A comprehensive graphical representation of the OFDM system model, whereby we can note the existence of interference-free parallel sub-channels in the frequency domain is shown below-

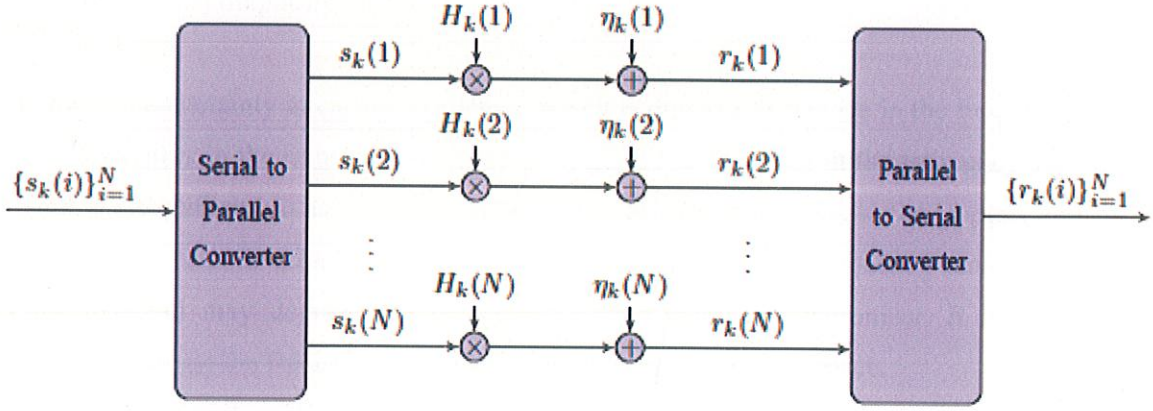

Fig16: Effective OFDM block showing interference-free paths with a cyclic prefix exceeding the CIR length

After the transmitter and the receiver blocks we add an additional block to calculate and compare the BERs of the two end signals.

CHAPTER 3

Carrier Frequency Offset Estimation

3.1 Carrier Frequency Offset:-

The sensitivity of OFDM systems to frequency offset compared with single carrier systems is a major disadvantage. In general, frequency offset is defined as the difference between the nominal frequency and actual output frequency.

In OFDM, the uncertainty in carrier frequency, which is due to a difference in the frequencies of the local oscillators in the transmitter and receiver, gives rise to a shift in the frequency domain. This shift is also referred to as frequency offset. It can also be caused due to the Doppler shift in the channel. The demodulation of a signal with an offset in the carrier frequency can cause large bit error rate and may degrade the performance of a symbol synchronizer. It is therefore important to estimate the frequency offset and minimize/eliminate its impact.

If frequency offset is denoted as f_Δ, the OFDM signal generated by the transmitter denoted as s(t) and y(t) is the signal received by the receiver, then

$$s(t) = e^{jwt}\, x(t)$$

$$y(t) = e^{j(w-w')t}\, x(t)$$

$$\Delta w = w - w' = 2\pi f_\Delta$$

Then the received signal has phase offset equal to-

$$Y(nT) = e^{j\Delta wnT} x(nT)$$

$$\Phi(n) = \Delta wnT$$

The frequency response of each sub-channel should be zero at all other sub-carrier frequencies, i.e., the sub-channels shouldn't interfere with each other. The effect of frequency offset is a translation of these frequency responses resulting in loss of orthogonality between the sub-carriers and leading to ICI. Thus if f_{tx} and f_{rx} denote the carrier frequencies in the transmitter and receiver, respectively.

$$(f_{\Delta} = f_{tx} - f_{rx})$$

3.2 Effect of Carrier Frequency Offset on Baseband signal:-

The baseband transmit signal is converted up to the pass band by a carrier modulation and then converted down to the baseband by using a local carrier signal of the same carrier frequency at the receiver. In general, there are two types of distortion associated with the carrier signal. One is the phase noise due to the instability of carrier signal generators used at the transmitter and receiver. The other is the carrier frequency offset (CFO) caused by Doppler frequency shift fd. Furthermore, even to generate exactly the same carrier frequencies in the transmitter and receiver, there may be an unavoidable difference between them due to the physically inherent nature of the oscillators. Let f_c and f_c' denote the carrier frequencies in the transmitter and receiver, respectively. Let f_{Δ} denote their difference (i.e., $f_{\Delta} = f_c - f_c'$). Meanwhile, Doppler frequency fd is determined by the carrier frequency fc and the velocity v of the terminal (receiver) as-

$$f_d = \frac{v.fc}{c}$$

where c is the speed of light. Let us define the normalized CFO, ε as a ratio of the CFO to subcarrier spacing Δf , shown as-

$$\varepsilon = f_{\Delta}/\Delta f$$

Let ε_i and ε_f denote the integer part and fractional part of ε, respectively, and therefore, $\varepsilon = \varepsilon_i + \varepsilon_f$, where $\varepsilon_i = [\varepsilon]$. For the time-domain signal x[n], a CFO of ε causes a phase offset of $2\pi n\varepsilon$, that is, proportional to the CFO ε and time index n. Note that it is equivalent to a frequency shift of $-\varepsilon$ on the frequency-domain signal X[k].

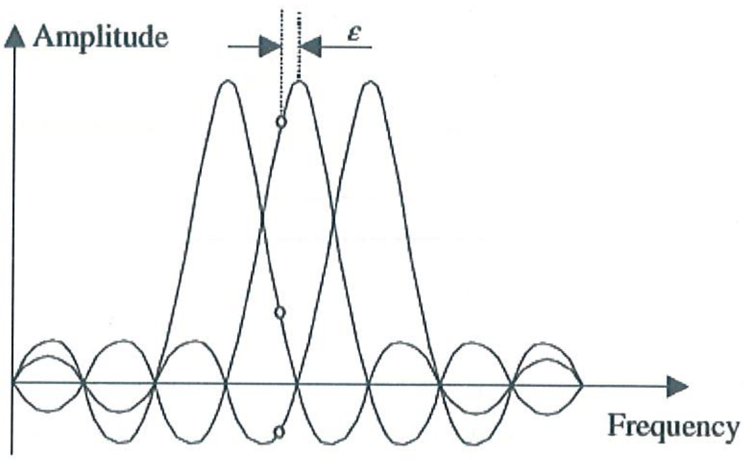

Fig 17: Inter Carrier Interference (ICI) subject to CFO

Fig. 17 shows that the frequency shift of $-\varepsilon$ in the frequency-domain signal X[k] subjects to the CFO of ε and leads to an inter-carrier interference (ICI), which means a subcarrier frequency component is affected by other subcarrier frequency components. To look into the effect of CFO, we assume that only a CFO of ε exists between transmitter and receiver, without any phase noise.

In a typical wireless communication system, the signal to be transmitted is up converted to a carrier frequency prior to transmission. The receiver is expected to tune to the same carrier frequency for down converting the signal to baseband, prior to demodulation.

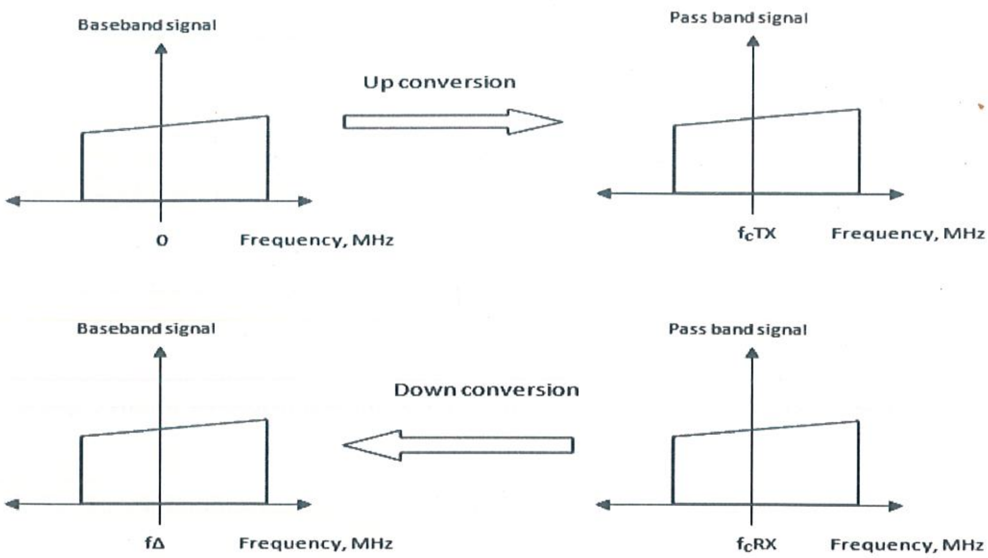

Fig 18: Up/down conversion

However, due to device impairments the carrier frequency of the receiver need not be same as the carrier frequency of the transmitter. When this happens, the received baseband signal, instead of being centered at DC (0MHz), will be centered at a frequency f_Δ, where.

$$f_\Delta = f_{Tx} - f_{Rx}$$

The baseband representation is (ignoring noise),

$$y(t) = x(t)e^{j2\pi f_\Delta t}, \text{ where}$$

$y(t)$ is the received signal

$x(t)$ is the transmitted signal and

f_Δ is the frequency offset.

3.3 CFO Estimation Techniques:-

Like Synchronization timing offset, Carrier Frequency Offset can also be estimated in time domain as well as frequency domain.

3.3.1 Using cyclic prefix (CP):

With perfect symbol synchronization, a CFO of ε results in a phase rotation of $2\pi n\varepsilon/N$ in the received signal. Under the assumption of negligible channel effect, the phase difference between CP and the corresponding rear part of an OFDM symbol (spaced N samples apart) caused by CFO ε is $2\pi N\varepsilon/N = 2\pi\varepsilon$. Then, the CFO can be found from the phase angle of the product of CP and the corresponding rear part of an OFDM symbol, for example, $\hat{\varepsilon}=(1/2\pi)\arg\{y_l^*[n]y_l[n+N]\}$, $n=-1, -2, \ldots, -N_g$. In order to reduce the noise effect, its average can be taken over the samples in a CP interval as-

$$\hat{\varepsilon} = \frac{1}{2\pi}\arg\{ \sum_{n=-N_G}^{-1} y_l *[n] y_l[n+N]\}$$

Since the argument operation arg() is performed by using $\tan^{-1}()$, the range of CFO estimation is $[-\pi,+\pi)/2\pi = [-0.5,+0.5)$ so that $|\hat{\varepsilon}| < 0{:}5$ and consequently, integral CFO cannot be estimated by this technique.

Note that $y_l^*[n]\, y_l[n+N]$ becomes real only when there is no frequency offset. This implies that it becomes imaginary as long as the CFO exists. In fact, the imaginary part of $y_l^*[n]\, y_l[n+N]$ can be used for CFO estimation. In this case, the estimation error is defined as-

$$e_\varepsilon = \frac{1}{L}\sum_{n=1}^{L} \mathrm{Im}\{y_l *[n] y_l[n+N]\}$$

where L denotes the number of samples used for averaging.

3.3.2 Using Preamble:

From the IEEE 802.11a specifications, it can be observed that each OFDM packet has a preamble structure formed using 10 short preambles of duration $0.8\mu s$ each. This short preamble is constructed by defining 12 subcarriers only (out of the available 52 subcarriers) where the modulation of individual subcarriers ensures a low **peak to average power ratio (PAPR)**.

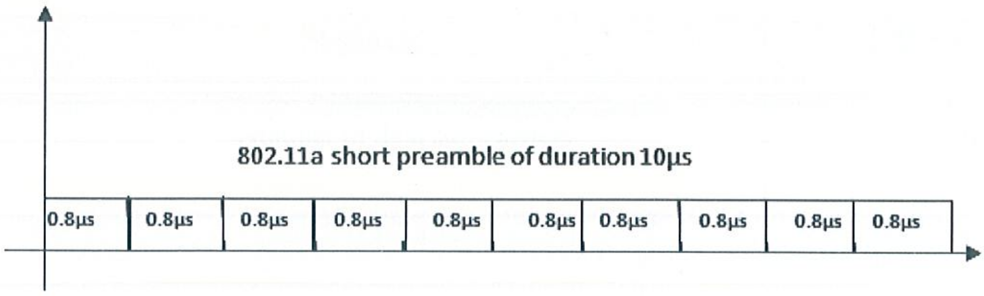

Fig 19: OFDM short preamble 802.11a specification

From the equation defined in the previous section,

$$y(t) = x(t)e^{j2\pi f_{\Delta}t}$$

Given that short preamble is periodic with $\delta t = 0.8\mu s$,

$$y(t-\delta t) = x(t)e^{j2\pi f_{\Delta}(t-\delta t)}$$

At the receiver as both $y(t)$ and $y(t-\delta t)$ are known,

Taking angle() of both sides of the equation

So, the frequency offset f_{Δ} is- $f_{\Delta} = -\frac{\Lambda y(t-\delta t)y*(t)}{2\pi\delta t}$

CHAPTER 4

Simulation Results

4.1 Simulation parameters:-

S.No.	Parameters	Value
1.	Modulation Used	BPSK, QPSK
2.	Number of data sub-carriers	64
3.	IFFT size	64
4.	Guard Period Type	Cyclic extension
5.	Cyclic prefix length	16
6.	Channel used	AWGN
7.	Bandwidth	20MHz
8.	Sub-carrier frequency spacing	20/64 = 0.3125 MHz
9.	T_{FFT}: IFFT/FFT period	3.2μsec
10.	T_{cp}: cyclic prefix duration	0.8μsec
11.	Total OFDM symbol duration	$T_{FFT} + T_{cp} = 4\mu sec$

4.2 Results:-

1. A plot of SNR vs. BER of the received signal is obtained from the code. This semilogy graph represents the variation of the bit error rate with varying values of signal to noise ratio when passed through an awgn channel.

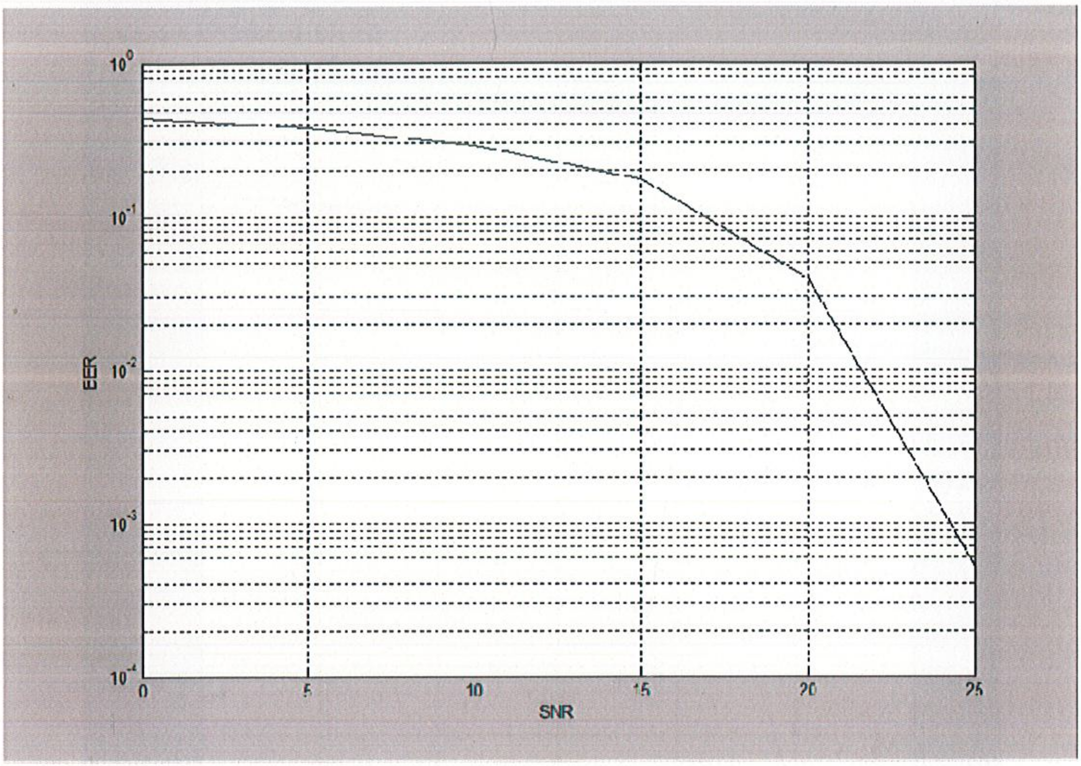

Fig 20: BER of the OFDM signal passed through AWGN channel

SNR	BER
0	0.4263
5	0.3756
10	0.2900
15	0.1609
20	0.0398

The above graph between BER and SNR shows that BER is a monotonically decreasing function with increasing SNR.

2 After the transmission of the signal through the channel, it is passed through the frequency estimator (using preamble technique) and then the corrector and then the BER is obtained of the resulting signal. It can be observed that the BER (with CFO estimator) is much better than BER (without CFO estimator). The modulation technique used is BPSK.

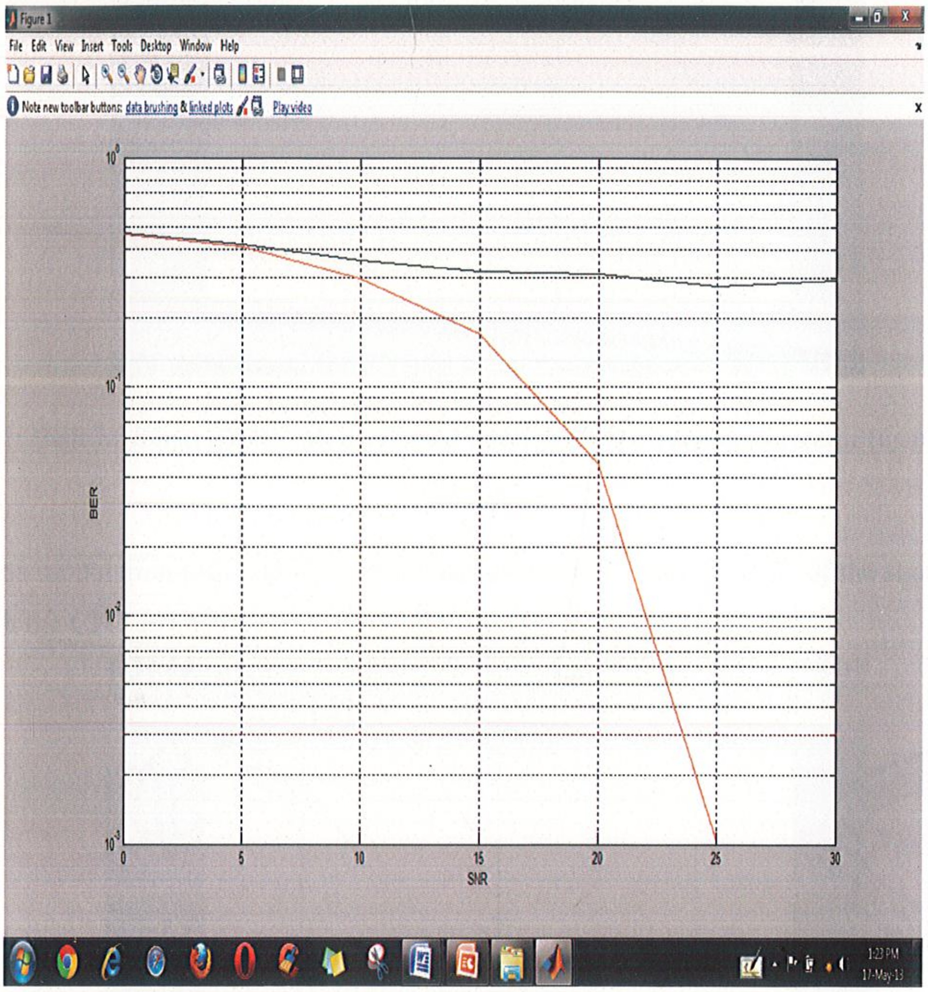

Fig 21: BER of signal (BPSK Mod.) passed through CFO estimator/corrector (using preamble technique)

3 Now the same process is repeated for Cyclic Prefix method.

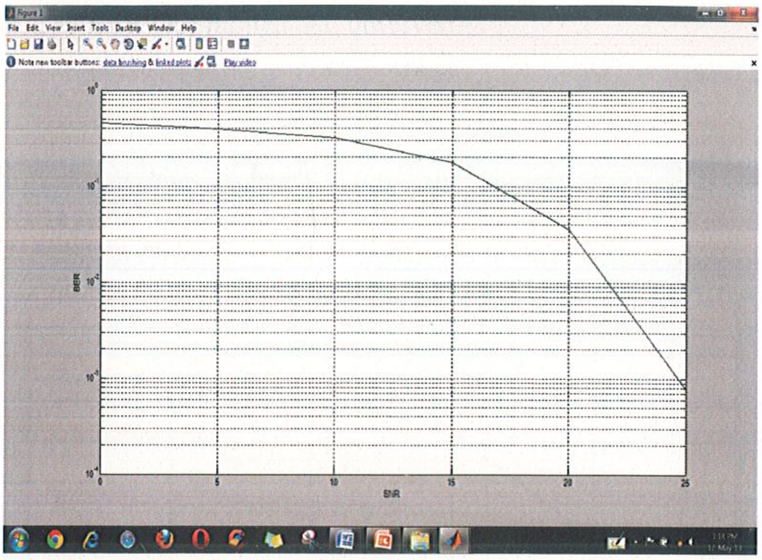

Fig 22: BER of signal (BPSK Mod.) passed through CFO estimator/corrector (using CP technique)

4 Now the modulation technique has been changed to QPSK and the BER of the signal passed through the CFO estimator/corrector (with Preamble technique) is plotted.

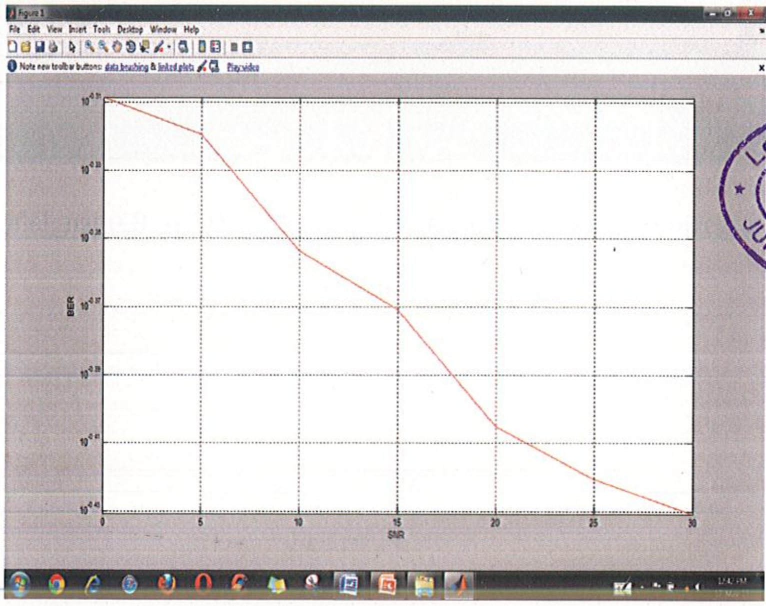

Fig 23: BER of signal (QPSK Mod.) passed through CFO estimator/corrector (using Preamble technique)

5 Again the process is repeated with same QPSK mod. But this time CFO estimator uses cyclic prefix technique. The BER was plotted and observed-

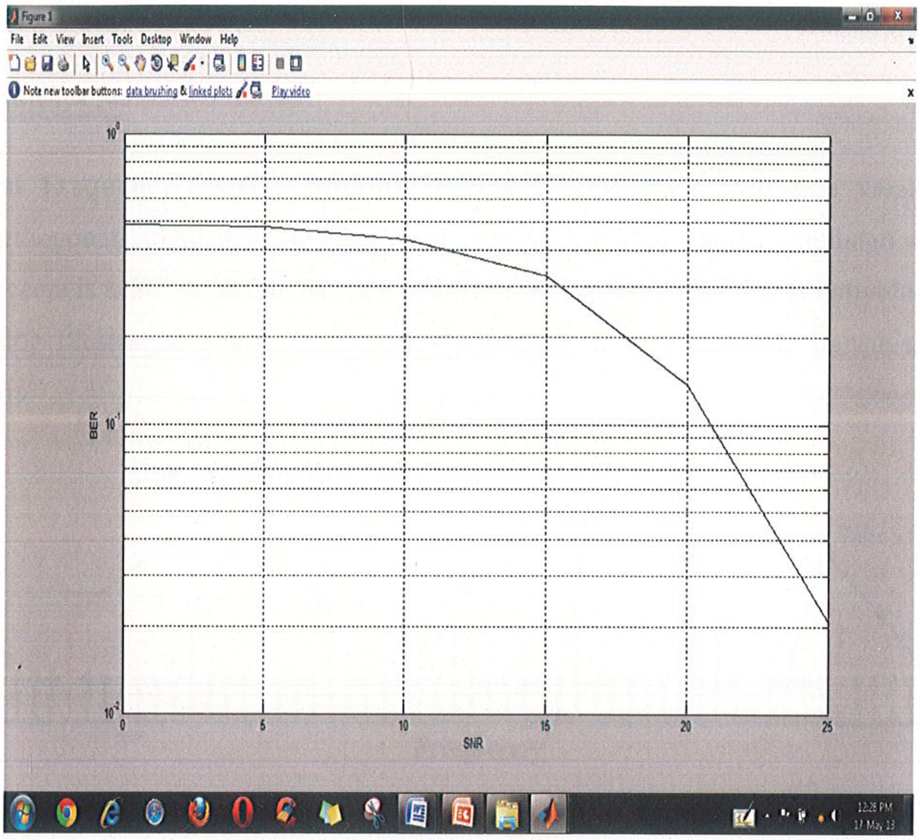

Fig 24: BER of signal (QPSK Mod.) passed through CFO estimator/corrector (using CP technique)

CHAPTER 5

APPLICATIONS OF OFDM SYSTEMS

5.1 OFDMA:-

Orthogonal Frequency-Division Multiple Access (OFDMA)is a multi-user version of the popular orthogonal frequency division multiplexing (OFDM) digital modulation scheme .Multiple access is achieved in OFDMA by assigning subsets of subcarriers to individual users as shown in the illustration below. This allows simultaneous low data rate transmission from several users.

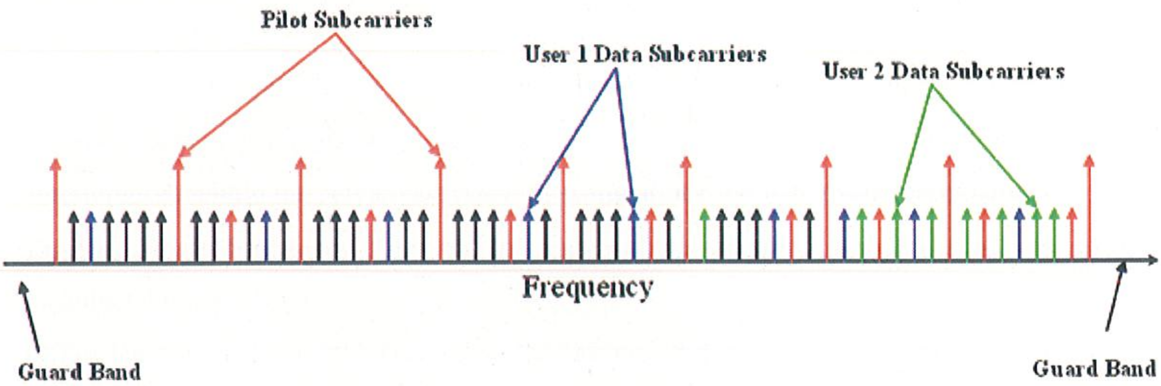

Fig 25: OFDMA (A multi-user version of OFDM)

5.1.1 Advantages over CDMA:

- OFDM can combat multipath interference with more robustness and less complexity.
- OFDMA can achieve a higher MIMO spectral efficiency due to providing flatter frequency channels than a CDMA rake receiver can.
- No cell size breathing as more users connect.

5.1.2 Advantages over OFDM with time-domain statistical multiplexing:

- Allows simultaneous low-data-rate transmission from several users.
- Pulsed carrier can be avoided.
- Lower maximum transmission power for low data rate users.
- Shorter delay and constant delay.
- Contention-based multiple access (collision avoidance) is simplified.
- Further improves OFDM robustness to fading and interference.
- Combat narrow-band interference.

5.1.3 OFDMA Advantages:

- Flexibility of deployment across various frequency bands with little needed modification to the air interface.
- Averaging interferences from neighboring cells, by using different basic carrier permutations between users in different cells.
- Interferences within the cell are averaged by using allocation with cyclic permutations.
- Enables Single Frequency Network coverage, where coverage problem exists and gives excellent coverage.
- Offers Frequency diversity by spreading the carriers all over the used spectrum.
- Allows per channel or per sub channel power

5.1.4 Disadvantages of OFDMA:

- Higher sensitivity to frequency offsets and phase noise.
- Asynchronous data communication services such as web access are characterized by short communication bursts at high data rate. Few users in a base station cell are transferring data simultaneously at low constant data rate.

- The complex OFDM electronics, including the FFT algorithm and forward error correction, are constantly active independent of the data rate, which is inefficient from power consumption point of view, while OFDM combined with data packet scheduling may allow FFT algorithm to hibernate during certain time intervals.
- The OFDM diversity gain, and resistance to frequency-selective fading, may partly be lost if very few sub-carriers are assigned to each user, and if the same carrier is used in every OFDM symbol. Adaptive sub-carrier assignment based on fast feedback information about the channel, or sub-carrier frequency hopping, is therefore desirable.
- Dealing with co-channel interference from nearby cells is more complex in OFDM than in CDMA. It would require dynamic channel allocation with advanced coordination among adjacent base stations.
- The fast channel feedback information and adaptive sub-carrier assignment is more complex than CDMA fast power control.

5.1.5 Usage:

OFDMA is used in:

- the mobility mode of the IEEE 802.16 Wireless MAN standard, commonly referred to as WiMAX,
- the IEEE 802.20 mobile Wireless MAN standard, commonly referred to as MBWA,
- MoCA 2.0,
- the downlink of the 3GPP Long Term Evolution (LTE) fourth generation mobile broadband standard. The radio interface was formerly named **High Speed OFDM Packet Access** (HSOPA), now named Evolved UMTS Terrestrial Radio Access (E-UTRA).
- the Qualcomm Flarion Technologies Mobile Flash-OFDM
- the now defunct Qualcomm/3GPP2 Ultra Mobile Broadband (UMB) project, intended as a successor of CDMA2000, but replaced by LTE.

- OFDMA is also a candidate access method for the IEEE 802.22 **Wireless Regional Area Networks** (WRAN). The project aims at designing the first cognitive radio based standard operating in the VHF-low UHF spectrum (TV spectrum).

...

CONCLUSION

So far it has been seen that there are many advantages of using an OFDM system like it can be applied to the CDMA systems, where codes are used to make data sequences independent (also orthogonal) which allows many independent users to transmit in same space successfully so it provides us with multi-user diversity where different users may perceive different channel qualities, but a deep faded channel for one user may still be favorable to others. Besides this, the receiver of OFDM system is simple as it eliminates the intra-cell interference avoiding CDMA type of multi-user detection; the orthogonality of code is destroyed by selective fading and only FFT processor is required.

The effect of offset is extremely detrimental to the efficiency of OFDM systems, therefore there is a dire need to correct the same.The frequency offset can be estimated and corrected using various algorithms, two of which are (CP based and Preamble based) implemented using matlab.

Also there are disadvantages to them like tight synchronization between users are required for FFT in receiver and pilot signals have to be used for synchronizations. Apart from this there is co-channel interference for which dynamic channel allocation with advanced coordination among adjacent base stations has to be done.

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BOOKS USED:

MIMO-OFDM Wireless Communications With MATLAB – Wiley

APPENDIX

MATLAB Implementation:-

Basic OFDM structure:

```
clc;
clear all;
close all;
m=input('enter the parameter M of the signal matrix: ');
n=input('enter the 45arameter N of the signal matrix: ');
N=m*n;
s=randint(m,n,2);
display(s);

%series to parallel%
S1=reshape(s,N,1);
display(S1);

z=pskmod

x=ifft(z,N);

%parallel to series%
S2=reshape(x,1,N);
display(S2);

%adding cyclic prefix%
v=(N)/4;
for w1=1:v
y(w1)=S2(N-v+w1);
end
for w2=1:N
y(v+w2)=S2(w2);
end

display(y);


R2= [0 0 0 0 0 0 0];
for q2=0:1:30
w=1;
```

```
%awgn channel%
for w3=0:5:30
Q1(w)=w3;
    a=awgn(y,w3);


%removing cyclic prefix%
fori=1:N
c(i)=a(i+v);
end

%series to parallel%
S3=reshape(c,N,1);

%fft%
F=fft(S3,N);

Z1=(pskdemod(F,4));
%parallel to series%
S4=reshape(Z1,1,N);
display(S4);

%bit error%
[NUMBER,RATIO]=biterr(S1,Z1);

display(NUMBER);
display(RATIO);
R1(w)=RATIO;
R2(w)=R2(w)+R1(w);
w=w+1;
end
end

for w4=1:1:(w-1)
R2(w4)=(R2(w4)/q2);
end
semilogy(Q1,R2,'r');
xlabel('SNR');
ylabel('BER');
grid on;
```

Estimation Implementation:

- Technique using cyclic prefix:

```
of2=0;
for w5=1:v
cj=conj(a(v+w5));
    of1=fft(a(w5))*fft(cj);
    of2=of2+of1;
display(of2);
end
display(of2);
an=(angle(of2))/(v*2*3.14);
display(an);

  %correcting%
for w6=1:1:v
f1(w6)=a(w6)-an;
end
for w7=v:N+v
f1(w7)=a(w7);
end
display(f1);
```

- Technique using preamble:

```
for w5=1:1:(N+v-1)
cj(w5)=conj(a(w5+1));
of=fft(a(w5))*fft(cj);
display(of);
an=angle(of)/(2*3.14);
display(an);
end


%offset correction%
for w6=1:1:(N+v-1)
f1(w6)=a(w6)-an(w6);

end
f1(N+v)=a(80);
display(f1);
```