Jaypee University of Information Technology, Waknaghat

TEST-3 Examination - December 2024

B.Tech - VII Semester (ALL)

Course Code/Credits: 22B1WMA731/3 | Max. Marks: 35
Course Title: Linear Algebra for Data Science & Machine Learning
Course Instructor: RAD | Max. Time: 2 Hours

Note: (a) ALL questions are compulsory.
(b) Scientific calculators are allowed.
(c) The candidate is allowed to make suitable numeric assumptions wherever required.

Q.No	Question	CO	Marks
Q1	Which of the sets are vector (sub)spaces? Justify your answer. (a) $\mathbf{U} = \left\{ \begin{bmatrix} a+2 \\ 2a \end{bmatrix} : a \in \mathbb{R} \right\}$ (b) $\mathbf{V} = \left\{ \begin{bmatrix} a \\ b \\ c \end{bmatrix} : a, b, c \in \mathbb{R},\ a - 2b = 0,\ a - 3b + 2c = 0 \right\}$	CO-1	4
Q2	Prove that $\mathbf{w} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \in \mathbb{R}^3$ is a linear combination of the vectors: $v_1 = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \quad v_2 = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}, \quad v_2 = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}.$	CO-1	4
Q3	Apply Gram-Schmidt orthogonalization process to the following vectors to obtain first two orthogonal vectors: $\begin{pmatrix} -1 \\ 1 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 3 \\ -1 \\ 3 \end{pmatrix}, \begin{pmatrix} 1 \\ 3 \\ 5 \\ 7 \end{pmatrix}.$	CO-2	5
Q4	Consider the following 3×2 rectangular matrix: $\mathbf{A} = \begin{bmatrix} 2 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$ (a) Find the singular values of **A**. (b) Compute the SVD factorization of **A**.	CO-2	6

Q.No	Question	CO	Marks
Q5	Perform the *QR decomposition* on the given the matrix: $$\mathbf{B} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \\ 1 & 1 \end{bmatrix}$$ The orthonormal matrix $\mathbf{Q}$ and the upper triangular matrix $\mathbf{R}$ satisfy $\mathbf{B} = \mathbf{QR}$. What is the first column of $\mathbf{Q}$? Justify your answer. (a) $\frac{1}{\sqrt{3}}[\,1\ \ 1\ \ 1\,]^{\mathbf{T}}$ (c) $\frac{1}{\sqrt{2}}[\,1\ \ -1\ \ 0\,]^{\mathbf{T}}$ (b) $\frac{1}{\sqrt{2}}[\,1\ \ 1\ \ 0\,]^{\mathbf{T}}$ (d) $\frac{1}{\sqrt{3}}[\,1\ \ 0\ \ -1\,]^{\mathbf{T}}$	CO-2	3
Q6	A real estate company wants to predict housing prices based on the size of the living area. The company uses historical data of house prices (in \$1000s) and living areas (in sq. ft.) to train the model. The training data is as follows: **Living Area (x)** / **Price (y)**: 1000 / 150; 1500 / 200; 2000 / 250 The company's objective is to learn the relationship between the living area x and the price y using linear regression with gradient descent. The model being used is $\hat{y} = \theta_0 + \theta_1 x$. Take $\alpha = 0.01$. (a) Define the *cost function* for the problem. (b) Perform first iteration of *gradient descent* with $\theta_0 = 0, \theta_1 = 0$.	CO-4	5
Q7	A marketing agency collects data on monthly spending (in thousands of dollars) of two customers, on two advertising platforms: Facebook Ads and Google Ads. The data matrix is as follows: *Features* / Spends for Customer 1 / Spends for Customer 2: Google / 4 / 8; Facebook / 10 / 5 The goal is to reduce the dataset to a lower-dimensional representation while retaining the majority of the variance. This helps the agency analyze spending patterns across platforms efficiently. Perform *principal component analysis* on this dataset to answer: (a) Calculate the covariance matrix $\mathbf{S}$ of the centered data. (b) Compute the eigenvalues & eigenvectors of $\mathbf{S}$. (d) Transform the data onto the first principal component (PC1) (c) What proportion of the total variance is captured by PC1?	CO-3	8