

Jaypee University of Information Technology, Waknaghat

Make-up Examination, April 2018

B.Tech (ECE/CSE/IT) - Semester IV

Course Title: Probability Theory and Random Processes

Max. Marks: 25 marks

Course Code: 10B11MA411

Course Credits: 4

Max. Time: 90 minutes

**Note:** Answer all the questions. Define random variables along with range where applicable.

Scientific calculators are allowed. Necessary statistical tables are supplied.

1. A diagnostic test has a probability 0.95 of giving a positive result when applied to a person suffering from a certain disease, and a probability 0.10 of giving a 'false positive' when applied to a non-sufferer. It is estimated that 0.5% of the population are sufferers. Suppose that the test is now administered to a person about whom we have no relevant information relating to the disease (apart from the fact that he/she comes from this population). (4 Marks)

- (a) Find the probability that the test result will be positive.  
(b) Given a positive result, find the probability that the person is a sufferer.

2. A supplier of kerosene has a weekly demand  $Y$  possessing density function given by (4 Marks)

$$f(y) = \begin{cases} y & , \quad 0 \leq x \leq 1 \\ 1 & , \quad 1 \leq x < 1.5 \\ 0 & , \quad \text{elsewhere} \end{cases}$$

with measurements in hundreds of gallons. Find *cumulative distribution function* of  $Y$ .

3. Consider the following joint density of  $X$  and  $Y$  (4 Marks)

$$f(x, y) = \begin{cases} 2 & , \quad 0 \leq y \leq x \leq 1 \\ 0 & , \quad \text{otherwise} \end{cases}$$

- (a) Find the marginal density of  $Y$ .  
(b) Find  $P(1/4 < X < 1/2 \mid Y = 1/4)$ .

4. The joint probability density function of  $X$  and  $Y$  is (5 Marks)

$$g(x, y) = \begin{cases} 2e^{-x-y} & , \quad 0 \leq x \leq y < \infty \\ 0 & , \quad \text{otherwise} \end{cases}$$

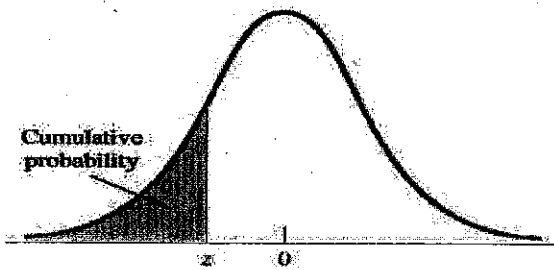
Define new random variables by  $U = Y - X$  and  $V = \sqrt{X}$ . Find joint density of  $U$  and  $V$ .

5. A sender sends messages to a receiver over a wireless channel. Each message is transmitted 4 times. A single transmission of a message is successful with probability 0.9. Among 4 transmissions for a message, find probability that at least 3 transmissions are successful? (3 Marks)

6. Let  $X = IQ$  of an individual and is normally distributed with a mean of 100 and a standard deviation of 15. Suppose one individual is randomly chosen. (5 Marks)

- (a) Find the probability that the person has an  $IQ$  greater than 120.  
(b) Mensa is an organization whose members have the top 2% of all  $IQ$ s. Find the minimum  $IQ$  needed to qualify for the Mensa organization.

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z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
-3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
-3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
-3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
-2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
-1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
-0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
-0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
-0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
-0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
-0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
-0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641