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JAYPEE UNIVERSITY OF INFORMATION TECHNOLOGY, WAKNAGHAT
SUMMER SEMSTER (JUNE – JULY 2018)
MID TERM EXAMINATION

COURSE CODE: 10B11MA411 MAX. MARKS: 50
COURSE NAME: Probability Theory & Random Processes
COURSE CREDITS: 4 MAX. TIME: 2 Hr

***Note:** All questions are compulsory. Carrying of mobile phone during examinations will be treated as case of unfair means. Use of scientific calculator is allowed.*

1. Answer the following questions: (2x5 = 10 Marks)
   (a) What is the expected number of throws of a die to get first 6?
   (b) What is $P(A\ XOR\ B)$?
   (c) If $A \subseteq B$ then $P(B \mid A) =$ ______________.
   (d) State Baye's theorem.
   (e) If it true that if A and B are independent events, then $\overline{A}$ and $\overline{B}$ are also independent? Justify your answer with proof.
2. Consider the joint probability density function of X and Y: (7)

$$f(x,y) = \begin{cases} \frac{1}{8}x(x-y), & 0 < x < 2 \ \ and \ \ -x < y < x \\ 0, & otherwise \end{cases}$$

   (a) Find the marginal density function $f_X(x)$. (b) Determine the conditional density function $f_{Y|X}(y \mid x)$.
3. Let X and Y be continuous random variables having joint density function (8)

$$f(x,y) = \begin{cases} c(x^2 + y^2) & 0 \le x \le 1,\ 0 \le y \le 1 \\ 0, & otherwise \end{cases}$$

   Determine (a) the constant c (b) $P(X < 1/2, Y > 1/2)$ (c) $P(Y < 1/2)$
   (d) whether X and Y are independent.
4. An urn contains 10 white and 3 black balls. Another urn contains 3 white and 5 black balls. 2 balls are drawn at random from the first urn and placed in the second urn and then 1 ball is drawn at random from the second urn. What is the probability that it is white? (5)
5. If X and Y be independent random variables with probability density functions e^{-x}, $x \ge 0$ and e^{-y}, $y \ge 0$, find the PDF of $U = X/(X+Y)$ and $V = X + Y$. Are they independent? (6)
6. If (X, Y) is uniformly distributed 2D random variable over the triangular region R bounded by $y = 0$, $x = 3$, and $3y = 4x$. (6)
   Find $f_X(x), f_Y(x), E(X), E(Y), Var(X), Var(Y), Cov(X,Y), \rho(X,Y)$.
7. Assume that on the average one telephone number out of fifteen called between 2 pm and 3 pm on a week days is busy. What is the probability that if 6 randomly selected telephone numbers are called (a) Not more than three (b) at least three of them will be busy? (4)
8. In an urn there are r red balls and b blue balls. Balls are selected at random with replacement so that the first blue ball is obtained. What is the probability that exactly k draws needed for the first blue ball? What is the probability that at least m draws are needed for the first blue ball? (4)

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