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Test-I Examination, February 2018

B.Tech (ECE/CSE/IT)

Course Title: Probability Theory and Random Processes — Max. Marks: 15 marks
Course Code: 10B11MA411
Semester: IV — Max. Time: 1 hour

Note: Answer all the questions. ALL questions carry equal marks. Use of calculators is allowed.

1. Consider an urn with 10 type-I objects and 20 type-II objects, and an experiment involving sampling five objects. Find the probability of selecting 2 type-I objects are under the following cases:
   (a) Selection is done with replacement.
   (b) Selection is done without replacement.

2. A box contains three coins with a head on each side, four coins with a tail on each side, and two fair coins. One of these nine coins is selected at random and tossed once.
   (a) What is the probability that a head will be obtained?
   (b) Suppose that the toss does result in a head. What is the probability that the coin used for tossing was fair?

3. Let $\mathbf{X}$ be a discrete random variable whose *cumulative distribution function* is

$$\mathbb{F}(x) = \begin{cases} 0 & , \quad x < -3 \\ 1/6 & , \quad -3 \leq x < 6 \\ 1/2 & , \quad 6 \leq x < 10 \\ 1 & , \quad x \geq 10 \end{cases}.$$

   (a) Using the *distribution function*, find $\mathbb{P}(\mathbf{X} \leq 4)$ and $\mathbb{P}(-5 < \mathbf{X} \leq 4)$.
   (b) Obtain the *probability mass function* of $\mathbf{X}$.

4. Suppose that you have a fair 4-sided die, and let $\mathbf{X}$ be the random variable representing the value of the number rolled.
   (a) Write down the *moment generating function* for $\mathbf{X}$.
   (b) Use this *moment generating function* to compute the mean value of $\mathbf{X}$.

5. Consider the following joint density of $\mathbf{X}$ and $\mathbf{Y}$:

$$\mathtt{f}(x,\, y) = \begin{cases} 6e^{-(2x+3y)} & , \quad x \geq 0,\ y \geq 0 \\ 0 & , \quad \text{otherwise} \end{cases}$$

   (a) Determine the marginal density of $\mathbf{X}$.
   (b) Find $\mathbb{P}(\mathbf{X} < 1,\ \mathbf{Y} < 0.5)$.

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