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# Jaypee University of Information Technology, Waknaghat

## Test-3 Examination, May 2018

Course Title: Probability Theory and Random Processes  
Course Code: 10B11MA411  
Semester: IV

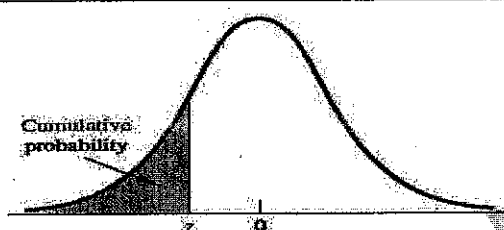
Program: B.Tech (ECE/CSE/IT)  
Marks: 35 marks  
Time: 2 hours

**Note:** Answer all the questions. Describe random variables along with range where applicable.  
Scientific calculators are allowed.

1. Suppose that 2 batteries are randomly chosen without replacement from the following group of 12 batteries: 3 new, 4 used (working), 5 defective. Let  $X$  denote the number of new batteries chosen and  $Y$  denote the number of used batteries chosen. (4 Marks) [CO2]
  - (a) Determine the joint pmf of  $(X, Y)$ .
  - (b) Find  $P[X + Y < 2]$ .
2. (a) Consider the *uncorrelated* random variables  $X$ ,  $Y$  and  $Z$  with zero mean and *standard deviations* 5, 12 and 9 respectively. Suppose that  $U = X + Y$  and  $V = Y + Z$ . Determine the *correlation coefficient* between  $U$  and  $V$ . (2 Marks) [CO3]
  - (b) Suppose that  $M_X(t) = (0.6 + 0.4e^t)^8$ . What is the mgf of  $3X + 2$ ? (2 Marks) [CO2]
3. An aptitude test for pilots requires a series of operations to be performed quickly. Suppose that time needed to complete the test is normally distributed with mean 90 minutes and standard deviation 20 minutes. (4 Marks) [CO3]
  - (a) Find the probability that it takes a randomly selected pilot between 95 and 100 minutes to complete the test.
  - (b) The top 5% of pilots get an honor certificate. How fast must a pilot complete the test to get the certificate?
4. The *time till failure*  $T$  (in years) of a device has the *density function*: (4 Marks) [CO2]
$$f(t) = \begin{cases} 3t^{-4} & , 1 < t < \infty \\ 0 & , \text{else} \end{cases}$$
  - (a) Find the *reliability* function.
  - (b) Determine the *hazard rate* function.
5. Customers arrive to a restaurant according to a Poisson process  $\mathcal{N}(t)$  with rate of arrival being 20 per hour. The restaurant opens for business at 11:00 am. Let  $X_n$  denote the time (in minutes) of the arrival of the  $n^{\text{th}}$  customer. (4 Marks) [CO4]
  - (a) Find the probability of having 20 customers in the restaurant at 11:12 am given that there were 18 customers at 11:07 am.
  - (b) Compute  $P(X_3 > 15)$ .
6. Consider the **WSS** process given by  $X(t) = 10 \cos(100t + \theta)$ , where  $\theta$  is uniformly distributed over  $(-\pi, \pi)$ . Suppose that  $R_{XX}(t, t + \tau) = 50 \cos(100\tau)$ . (4 Marks) [CO4]
  - (a) Determine the *time-averaged* correlation function of  $X(t)$ .

- (b) Can you say that  $\mathbf{X}(t)$  is *ergodic* in correlation? Justify your answer.
7. A computer device can be either in a busy mode (state 1) processing a task, or in an idle mode (state 2), when there are no tasks to process. Being in a busy mode, it can finish a task and enter an idle mode any minute with the probability 0.2. Being in an idle mode, it receives a new task any minute with the probability 0.1 and enters a busy mode. The initial state is idle. Let  $\mathbf{X}_n$  be the state of the device after  $n$  minutes. (6 Marks) [CO4]
- Write down the *transition probability matrix*.
  - Compute  $P(\mathbf{X}_2 = \text{'idle'}, \mathbf{X}_1 = \text{'busy'}, \mathbf{X}_0 = \text{'idle'})$ .
  - Find the *steady-state distribution* of  $\mathbf{X}_n$ .
8. Stochastic process  $\mathbf{X}(t)$  is **WSS** and Gaussian with  $\mathcal{R}_{\mathbf{X}\mathbf{X}}(\tau) = 4e^{-2|\tau|}$ . (5 Marks) [CO4]
- Find  $P[\mathbf{X}(t) \leq 3]$ .
  - Find  $E\{[\mathbf{X}(t+1) - \mathbf{X}(t-1)]^2\}$ .

(Standard) Normal probability table to compute  $P(\mathbf{Z} \leq z)$



$z$	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
-1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
-0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
-0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
-0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
-0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
-0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641