Jaypee University of Information Technology, Waknaghat

TEST-2 Examination - April 2025

B.Sc. IV Semester (Mathematics and Computing)

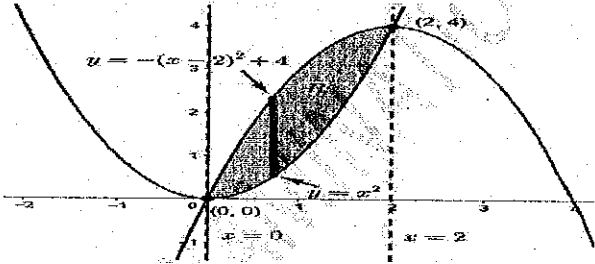Course Code/Credits: 24BS1MA412/3
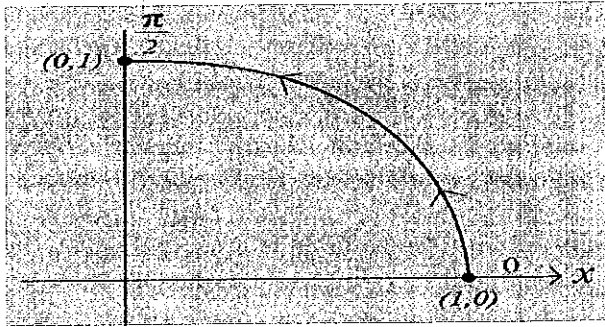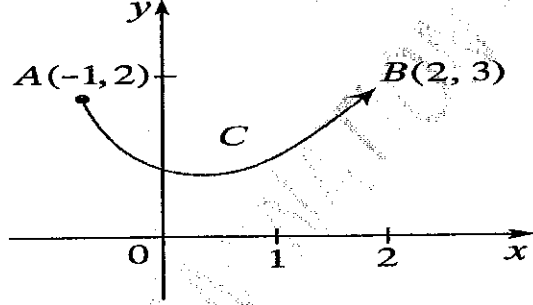
Course Title: Multivariable Calculus in Machine Learning

Course Instructors: RAD

Max. Marks: 25

Max. Time: 1 Hour 30 Min

Note: (a) ALL questions are compulsory.

(b) The candidate is allowed to make suitable numeric assumptions wherever required.

Q.No	Question	CO	Marks
Q1	Consider the function $f(x,y) = 2x^2 + y$: (a) Find the *gradient* of the function $f(x,y)$. (b) Find a *unit normal* to the *level* curve $f(x,y) = 1$ at $\mathcal{P}(-1,1)$.	CO-1	5
Q2	Consider the region bounded by curves $y = x^2$ and $y = -(x-2)^2+4$: (a) Set up the limits for the double integral $\iint_R 1\, dxdy$. (b) Use double integral to calculate the area of the shaded region.	CO-2	5
Q3	A vertical water tank is in the shape of a cylinder with height $h = 2$ meters and radius $r = 1$ meter. The density of water varies with depth due to temperature differences, modeled as $$\rho(x,y,z) = 1000 + 2z \quad \text{kg/m}^3$$ where z is the depth of water measured from bottom $(0 \le z \le 2)$. (a) Find the *total mass* $\int_0^2 \int_{-1}^1 \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} \rho(x,y,z)\ dx\,dy\,dz$ of the water in the tank. (b) Compute the *total weight* of the water in the tank in Newtons. (c) If the tank drains at a rate of 10 liters per second, how long will it take to empty completely?	CO-2	5

Q.No	Question	CO	Marks
Q4	Consider the force field $\mathbf{F}(x,y) = \langle x^2, -xy \rangle$ in moving a particle along the unit circular path from $(1,0)$ to $(0,1)$.	CO-3	5
	(a) Give the parameterization $\mathbf{r(t)} = x(t)\mathbf{i} + y(t)\mathbf{j}$ of the path.		
	(b) Find the *work done* by the *force field*.		
Q5	Consider the vector field $\mathbf{F}(x,y) = \langle 2xy - 3, x^2 + 4y^3 + 5 \rangle$.	CO-3	5
	(a) Show that $\mathbf{F}$ is *conservative* by showing that it is *gradient* of *potential* function $\mathbf{f}(x,y) = x^2 y - 3x + y^4 + 5y$.		
	(b) Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ along the curve C as shown in the figure.		

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