Jaypee University of Information Technology, Waknaghat

TEST-3 Examination - May 2025

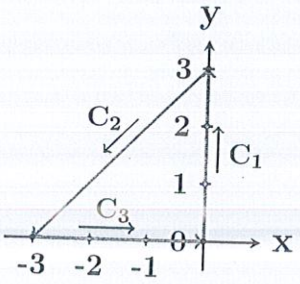
B.Sc. IV Semester (Mathematics and Computing)

Course Code/Credits: 24BS1MA412/3
Course Title: Multivariable Calculus in Machine Learning
Course Instructors: RAD

Max. Marks: 35
Max. Time: 2 Hours

Note: (a) ALL questions are compulsory.

(b) The candidate is allowed to make suitable numeric assumptions wherever required.

Q.No	Question	CO	Marks
Q1	Consider $f(x, y, z) = 2xy - z^2$. (a) Find the *directional derivative* at the point $(2, -1, 1)$ in the direction towards $(3, 1, -1)$. (b) In what direction is the *directional derivative* maximum?	CO-1	5
Q2	Consider the following double integral: $\int_0^4 \int_{-\sqrt{16-y^2}}^{\sqrt{16-y^2}} dx\, dy$ (a) Sketch the regions of integration. (b) Change the order of integration, but do not evaluate.	CO-2	5
Q3	Consider the following force field: $\mathbf{F}(x, y) = \langle y + \sin x, e^y - x \rangle$ acting on the particle traversing *counterclockwise* along the circle $x^2 + y^2 = 4$, starting and ending at point $(2, 0)$. (a) Parameterize the path representing the curve. (b) Calculate the work done on a particle by the force field.	CO-3	5
Q4	Consider the force field $\mathbf{F}(x,\ y) = \langle xy^2 + x^2,\ 4x - 1 \rangle$:  where $C := C_1 + C_2 + C_3$ is a closed curve oriented *countercockwise*. (a) Find the equation of line segment from $(0, 3)$ to $(-3, 0)$. (b) Use Green's Theorem to compute the line integral $\int_C \mathbf{F} \cdot \mathbf{dr}$.	CO-3	5

<table>
<tr><th>Q.No</th><th>Question</th><th>CO</th><th>Marks</th></tr>
<tr><td>Q5</td><td>Consider the function $f(x) = x^{2/3}(5+x)$.
(a) Identify the local extrema.
(b) Determine the points of inflection.
(c) Find the intervals where the function is convex.</td><td>CO-4</td><td>5</td></tr>
<tr><td>Q6</td><td>Consider the function $\mathtt{f}(x, y, z) = x + 2y + 3z$ on the curve of intersection of the plane $\mathtt{g}_1(x, y, z) = x - y + z = 1$ and the cylinder $\mathtt{g}_2(x, y, z) = x^2 + y^2 = 1$:
(a) Solve $\nabla\mathtt{f}(x, y, z) = \lambda\nabla\mathtt{g}_1(x, y, z) + \mu\nabla\mathtt{g}_2(x, y, z)$ for x, y, z.
(b) What is the maximum value of the function $\mathtt{f}(x,\ y,\ z)$?</td><td>CO-4</td><td>5</td></tr>
<tr><td>Q7</td><td>A fruit vendor wants to predict daily sales (in kilograms) based on the display area (in square meters) used for showcasing fruits. The vendor collects historical data for three days as shown below:
<table>
<tr><th>Display Area (x)</th><th>Sales (y)</th></tr>
<tr><td>2</td><td>4</td></tr>
<tr><td>3</td><td>6</td></tr>
<tr><td>4</td><td>8</td></tr>
</table>
The vendor's goal is to learn the relationship between the display area x and sales y using linear regression with gradient descent. The model used is $\hat{y} = \theta_0 + \theta_1 x$. Assume a learning rate $\alpha = 0.1$.
(a) Define the *cost function* for this problem.
(b) Perform first iteration of *gradient descent* with $\theta_0 = 0, \theta_1 = 1$.</td><td>CO-4</td><td>5</td></tr>
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