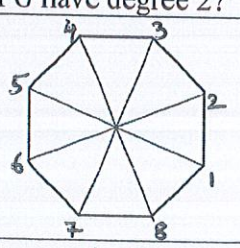COURSE CODE (CREDITS): 18B11CI414 (3) MAX. MARKS: 35
COURSE NAME: Discrete Computational Mathematics
COURSE INSTRUCTORS: RKB*, PKP MAX. TIME: 2 Hrs

Note: (a) All questions are compulsory. (b) The candidate is allowed to make suitable numeric assumptions wherever required for solving problems.

Q.No	Question	CO	Marks
Q1	Consider relations R and S defined on the set of integers $\mathbb{Z}$ as follows: $(a, b) \in R$ iff $a \equiv b \pmod 4$ & $(a, b) \in S$ iff $a \equiv b \pmod 6$. Compute $S \circ R$.	CO-3	3
Q2	Define a predicate with an example. Obtain the negation of the following quantified expression (over the discourse of real numbers $\mathbb{R}$): $\forall x \exists y \exists z \, (x^2 + y^2 + z^2 = 8)$. Explain whether the above quantified expression is a predicate or not?	CO-1	3
Q3	In a certain compression algorithm, the number of distinct binary strings of length n, denoted by a_n, that can be generated without containing two consecutive ones satisfies the recurrence relation: $$a_n = a_{n-1} + a_{n-2}; \quad n \geq 2, a_0 = 1 \ \& \ a_1 = 2.$$ Find a closed form expression using generating function.	CO-7	3
Q4	(a) Give an example of a cyclic group which is not abelian. (b) Find the generators in the group $(\mathbb{Z}_7, +_7, \times_7)$.	CO-6	2
Q5	Consider $(\mathbb{R}^+, *)$, here $\mathbb{R}^+$ denotes the set of all positive real numbers and operation $*$ is defined as $a * b = \frac{ab}{2}$ for all $a, b \in \mathbb{R}^+$. Verify whether $(\mathbb{R}^+, *)$ is an abelian group or not?	CO-6	3
Q6	Consider the algebraic structure $(3\mathbb{Z}, +, \cdot)$ where symbols are in their usual meaning. Verify whether given algebraic structure is a ring or not?	CO-6	3
Q7	Define an integral domain. Verify whether $(\mathbb{Q}, +, \cdot)$ is an integral domain or not?	CO-6	3
Q8	Let G be a bipartite graph with 22 vertices with partite set U and W, where $|U| = 12$. Suppose that every vertex in U has degree 3, while every vertex of W has degree 2 or 4. How many vertices of G have degree 2?	CO-4	3
Q9	State Kuratowski's theorem. Using the theorem, prove that the given graph with eight vertices is non-planar:	CO-4	3
Q10	(a) Find the number of edges in the complement of hypercube Q_4. (b) Give an example of a graph which is neither Eulerian nor Hamiltonian. (c) Write the general structure of adjacency matrix of a cycle graph C_n.	CO-4	3
Q11	A tree T with 50 end vertices has an equal number of vertices of degree 2, 3, 4 and 5. There is no vertex of degree greater than 5. What is number of vertices in the tree T?	CO-4	3
Q12	What is Euler's formula for a planar graph? Verify the formula for K_4. How many regions will be there in the planar representation of the connected graph G with 20 vertices, where every face (including the outer one) is bounded by exactly 3 edges?	CO-4	3