

Note: All questions are compulsory.

Q. No	Question	CO	Marks
Q1	<p>The accompanying Figure1 shows known flow rates of hydrocarbons into and out of a network of pipes at an oil refinery. Set up a linear system whose solution provides the unknown flow rates.</p> <p style="text-align: center;">Figure 1</p> <p style="text-align: center;">OR</p> <p>Find the reduced row echelon form of the augmented matrix for the linear system:</p> $\begin{aligned} 6x_1 + x_2 + 4x_4 &= -3 \\ -9x_1 + 2x_2 + 3x_3 - 8x_4 &= 1 \\ 7x_1 - 4x_3 + 5x_4 &= 2 \end{aligned}$ <p>Use your result to determine whether the system is consistent and, if so, find its solution.</p>	1	3
Q2	<p>Colors in print media, on computer monitors, and on television screens are implemented using what are called "colormodels". For example, in the RGB model, colors are created by mixing percentages of red (R), green (G), and blue (B), and in the YIQ model (used in TV broadcasting), colors are created by mixing percentages of luminance (Y) with percentages of a chrominance factor (I) and a chrominance factor (Q). The conversion from the RGB model to the YIQ model is accomplished by the matrix equation</p> $\begin{bmatrix} Y \\ I \\ Q \end{bmatrix} = \begin{bmatrix} .299 & .587 & .114 \\ .596 & -.275 & -.321 \\ .212 & -.523 & .311 \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}$ <p>What matrix would you use to convert the YIQ model to the RGB model?</p> <p style="text-align: center;">OR</p>	1	3

	Express the vector $u = (2, 3, 1, 2)$ in the form $u = w_1 + w_2$, where w_1 is a scalar multiple of $a = (-1, 0, 2, 1)$ and w_2 is orthogonal to a .		
Q3	<p>a) Show that if $S = \{v_1, v_2, v_3\}$ is a linearly independent set of vectors, then so are $\{v_1, v_2\}$, $\{v_1, v_3\}$, $\{v_2, v_3\}$, $\{v_1\}$, $\{v_2\}$, and $\{v_3\}$</p> <p>b) Find a basis for the row space and for the column space of the matrix.</p> $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$	2	1.5 + 1.5 = 3
Q4	<p>In part a) to f) determine whether the statement is true or false, and justify your answer.</p> <p>a) If A is a matrix with n columns, then the $\text{rank}(A) + \text{nullity}(A) = n$.</p> <p>b) If A is diagonalizable, then there is a unique matrix P such that $P^{-1}AP$ is diagonal.</p> <p>c) If $T: \mathbb{R}^n \rightarrow \mathbb{R}^n$ and $T(c_1x + c_2y) = c_1T(x) + c_2T(y)$ for all scalars c_1 and c_2 and all vectors x and y in \mathbb{R}^n, then T is a matrix transformation.</p> <p>d) The column space of a matrix A is the set of solutions of $Ax = b$.</p> <p>e) The matrix (given below) is a regular stochastic matrix.</p> $\begin{bmatrix} 0.2 & 1 \\ 0.8 & 0 \end{bmatrix}$ <p>f) The matrix (given below) represents reflection about a line.</p> $\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$	2	$0.5 * 6$ $= 3$
Q5	<p>Use the method of LU-decomposition</p> $\begin{bmatrix} 3 & -6 & -3 \\ 2 & 0 & 6 \\ -4 & 7 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 0 \\ 2 & 4 & 0 \\ -4 & -1 & 2 \end{bmatrix} \begin{bmatrix} 1 & -2 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$ <p>to solve the system:</p> $\begin{aligned} 3x_1 - 6x_2 - 3x_3 &= -3 \\ 2x_1 &+ 6x_3 &= -22 \\ -4x_1 + 7x_2 + 4x_3 &= 3 \end{aligned}$	3	3