

Course Code/Credits: 22B1WMA731/3

Course Title: Linear Algebra for Data Science & Machine Learning

Course Instructor: RAD

Max. Marks: 15

Max. Time: 1 Hour

Note: (a) ALL questions are compulsory.

(b) Scientific calculators are allowed.

(c) The candidate is allowed to make suitable numeric assumptions wherever required.

Q.No	Question	CO	Marks
Q1	<p>Answer the following:</p> <p>(a) A binary operation $*$ is defined on the set \mathbb{R} of real numbers by $x*y = \frac{1}{2} + x + y$, where $x, y \in \mathbb{R}$. Find the identity element of the operation.</p> <p>(b) In the group of non-zero rational numbers under the binary operation $*$ defined by $a*b = a + b + 1$, what is inverse of 2?</p>	CO-1	3
Q2	<p>Which of the following sets are vector subspaces? Justify answer.</p> <p>(a) $S_1 = \left\{ \begin{bmatrix} a \\ b \\ c \end{bmatrix} : a, b, c \in \mathbb{R}, a - 2b = 0, a - 3b + 2c = 0 \right\}$</p> <p>(b) $S_2 = \left\{ \begin{bmatrix} a & -1 \\ 0 & b \end{bmatrix} : a, b \in \mathbb{R} \right\}$</p>	CO-1	3
Q3	<p>Let \mathcal{P}_2 denote the vector space of all real polynomials of degree at most 2. Show that the set $\{f, g, h\}$ is linearly independent in \mathcal{P}_2:</p> <p>$f(x) = x^2 + 1, \quad g(x) = 3x - 1, \quad h(x) = -4x + 1.$</p>	CO-1	3
Q4	<p>Consider the subspace of the 4-dimensional vector space \mathbb{R}^4:</p> <p>$W = \left\{ (x_1, x_2, x_3, x_4) \in \mathbb{R}^4 \mid x_1 - x_2 + x_3 - x_4 = 0 \right\}.$</p> <p>Determine a basis and the dimension of W.</p>	CO-1	3
Q5	<p>Find the kernel of linear map $T : \mathbb{R}^5 \rightarrow \mathbb{R}^4$ defined by $T(\mathbf{x}) = A\mathbf{x}$:</p> $A = \begin{bmatrix} 1 & 2 & 0 & 1 & -1 \\ 2 & 1 & 3 & 1 & 0 \\ -1 & 0 & -2 & 0 & 1 \\ 0 & 0 & 0 & 2 & 8 \end{bmatrix}$	CO-1	3