JAYPEE UNIVERSITY OF INFORMATION TECHNOLOGY, WAKNAGHAT

TEST -1 EXAMINATION- 2025

B.Tech.-III Semester (CSE/IT/FSSD/AIDS/AIML/UXUI)

COURSE CODE (CREDITS): 25B11MA314 (4)  MAX. MARKS: 15

COURSE NAME: Mathematical Foundations for Artificial Intelligence and Data Science

COURSE INSTRUCTORS: RAD, BKP, SST  MAX. TIME: 1 Hour

**Note:** (a) *All questions are compulsory.*

(b) *The candidate is allowed to make suitable numeric assumptions wherever required for solving problems.*

(c) *Use of a scientific calculator is allowed.*

| Q. No. | Question | CO | Marks |
|---|---|---|---|
| Q1. | Determine whether the set $S = \{A \in M_{2\times2}(\mathbb{R}): det(A) = 0\}$, is a subspace of a vector space $(M_{2\times2}(\mathbb{R}), +, .)$. Illustrate your answer. | 1 | 3 |
| Q2. | a) Determine whether or not the vectors $(1,1,2)$, $(1,2,5)$ and $(5,3,4)$ form a basis for $\mathbb{R}^3$. <br> b) Find the $\ell_2$ norm of the vector $(-2, 6, 8, 5)$. | 2 | 2+1 |
| Q3. | Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be defined by $T(x,y) = (3x - y, 2x + 4y)$. Verify that $T$ is a linear transformation. | 1 | 3 |
| Q4. | Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation defined by $T(x,y) = (2x - y, x + 4y)$. Obtain the matrix representation of $T$ with respect to: <br> a) Standard basis, and <br> b) Basis, $B = \{(1,1), (2,-1)\}$. | 1 | 1+2 |
| Q5. | a) Find the kernel of the linear transformation $T: \mathbb{R}^3 \to \mathbb{R}^2$, defined by $$T(x) = \begin{bmatrix} 1 & -1 & -2 \\ -1 & 2 & 3 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}.$$ <br> b) Let $T: \mathbb{R}^5 \to \mathbb{R}^7$ be a linear transformation. What is the rank of $T$ if the nullity of $T$ is 4? | 1 | 2+1 |