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JAYPEE UNIVERSITY OF INFORMATION TECHNOLOGY, WAKNAGHAT
TEST -1 EXAMINATION- 2025
B.Tech.-III Semester (ECE/ECS/EEVLSI/MC)

COURSE CODE (CREDITS): 25B11MA311(4) MAX. MARKS: 15
COURSE NAME: Probability and Random Processes
COURSE INSTRUCTORS: SST MAX. TIME: 1 Hour

*Note: (a) All questions are compulsory.*

*(b) The candidate is allowed to make suitable numeric assumptions wherever required for solving problems.*

*(c) Use of a scientific calculator is allowed.*

<table>
<tr><th>Q. No.</th><th>Question</th><th>CO</th><th>Marks</th></tr>
<tr><td>Q1</td><td>A certain transistor is manufactured at three factories: Barnsley, Bradford, and Bristol. The Barnsley factory produces twice as many transistors as the Bradford factory; the Bradford and Bristol factories produce the same number of transistors. Experience shows that 0.2% of the transistors produced at Barnsley and Bradford are faulty, and so are 0.4% of those produced at Bristol are faulty. A service engineer finds a defective transistor while maintaining an electronic device. What is the probability that the transistor came from the Bradford factory?</td><td>1</td><td>3</td></tr>
<tr><td>Q2</td><td>Let $X$ and $Y$ be the number of hardware failures in two computer labs in a given month. The joint distribution of $X$ and $Y$ is given in the table below.
<table>
<tr><th colspan="2" rowspan="2">$p_{XY}(x,y)$</th><th colspan="3">$x$</th></tr>
<tr><th>0</th><th>1</th><th>2</th></tr>
<tr><td rowspan="3">$y$</td><td>0</td><td>0.52</td><td>0.20</td><td>0.04</td></tr>
<tr><td>1</td><td>0.14</td><td>0.02</td><td>0.01</td></tr>
<tr><td>2</td><td>0.06</td><td>0.01</td><td>0</td></tr>
</table>
a) Compute the probability of at least one hardware failure.<br>
b) Are $X$ and $Y$ independent? Why or why not?</td><td>2</td><td>1+2</td></tr>
<tr><td>Q3.</td><td>The waiting time, in hours, between successive speeders spotted by a radar unit is a continuous random variable with a cumulative distribution function:
$$F_X(x) = \begin{cases} 0, & x < 0 \\ 1 - e^{-8x}, & x \geq 0 \end{cases}$$
Find the probability of waiting less than 12 minutes between successive speeders</td><td>2</td><td>1.5+1.5</td></tr>
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<table>
<tr><td></td><td>a) using the cumulative distribution function of $X$;<br>b) using the probability density function of $X$.</td><td></td><td></td></tr>
<tr><td><b>Q4.</b></td><td>Suppose that the number of cars $X$ that pass through a car wash between 4:00 P.M. and 5:00 P.M. on any sunny Friday has the following probability distribution:
<table>
<tr><td>$x$</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td></tr>
<tr><td>$P(X = x)$</td><td>$\frac{1}{12}$</td><td>$\frac{1}{12}$</td><td>$\frac{1}{4}$</td><td>$\frac{1}{4}$</td><td>$\frac{1}{6}$</td><td>$\frac{1}{6}$</td></tr>
</table>
Let $g(X) = 2X - 1$ represent the amount of money, in dollars, paid to the attendant by the manager. Find the attendant's expected earnings for this particular time period.</td><td><b>2</b></td><td><b>3</b></td></tr>
<tr><td><b>Q5</b></td><td>The random variable $X$ has an exponential distribution:<br>$f_X(x) = \begin{cases} e^{-x}, x > 0 \\ 0, \quad else \end{cases}$, find the probability density function of $X^3$.</td><td><b>2</b></td><td><b>3</b></td></tr>
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