JAYPEE UNIVERSITY OF INFORMATION TECHNOLOGY, WAKNAGHAT

TEST – 2 EXAMINATION- 2025

B.Tech- III Semester (CSE/IT)

COURSE CODE (CREDITS): 24B11CI311 (03) MAX. MARKS: 25

COURSE NAME: COMPUTATIONAL FUNDAMENTALS FOR OPTIMIZATION

COURSE INSTRUCTORS: RBT, VSG MAX. TIME: 1 Hour 30 Min

Note: *(a) All questions are compulsory.*

(b) The candidate is allowed to make Suitable numeric assumptions wherever required for solving problems

Q.No	Question	CO	Marks
Q1	(a) Show that the function $f(x, y) = x^3 - 6xy - y^3$ has critical points at (0, 0) and (−2, 2). (b) Use the Hessian form of the second derivative test to show that f has a relative maximum at (−2, 2) and a saddle point at (0, 0). (c) Verify that the matrix is orthogonal. $\begin{bmatrix} \frac{3}{5} & -\frac{4}{5} \\ \frac{4}{5} & \frac{3}{5} \end{bmatrix}$	4	1 + 2 + 2
Q 2	(a) Distribution of heights of peoples follows binomial distribution (True / False). Justify your answer. (b) Suppose that the average number of accidents occurring weekly on a particular stretch of a highway equals 3. Calculate the probability that there is at least one accident this week. (c) Out of 800 families with 4 children each, how many families would be expected to have (i) 2 boys and 2 girls (ii) atmost 2 girls. Assume equal probabilities for boys and girls.	4	1 + 2 +2
Q3	(a) Let R^2 have the weighted Euclidean inner product $\langle u, v \rangle = 2 u1\, v1 + 3\ u2\, v2$ and let u = (1 , 1), v = (3 , 2) , w = (0 , -1) , and k = 3. Compute the stated quantities. $\langle \mathbf{u}, \mathbf{v} \rangle \quad \langle k\mathbf{v}, \mathbf{w} \rangle \quad d(\mathbf{u}, \mathbf{v}) \quad \lVert \mathbf{u} - k\mathbf{v} \rVert$ (b) In parts (i)–(vi) determine whether the statement is true or false,	2	2 + 3

	and justify your answer. (i) The dot product on R^2 is an example of a weighted inner product. (ii) The inner product of two vectors cannot be a negative real number. (iii) $\langle u, v + w \rangle = \langle v, u \rangle + \langle w, u \rangle$ (iv) $\langle ku, kv \rangle = k2 \langle u, v \rangle$ (v) If $\langle u, v \rangle = 0$, then $u = 0$ or $v = 0$. (vi) If A is an $n \times n$ matrix, then $\langle u, v \rangle = Au \cdot Av$ defines an inner product on R^n.		
Q 4	Find a singular value decomposition of A $$A = \begin{bmatrix} 4 & 6 \\ 0 & 4 \end{bmatrix}$$ **OR** Find the QR decomposition of the matrix $$A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$$	3	5
Q5	a) What conditions must $k1$ and $k2$ satisfy for $\langle \mathbf{u}, \mathbf{v} \rangle = k1\ u1\ v1 + k2\ u2\ v2$ to define an inner product on R^2? Justify your answer. b) Compute the standard inner product on M_{22} of the given matrices. $$U = \begin{bmatrix} 3 & -2 \\ 4 & 8 \end{bmatrix},\ V = \begin{bmatrix} -1 & 3 \\ 1 & 1 \end{bmatrix}$$ c) Let the vector space P_2 have the inner product. Find the following for $\mathbf{p} = p(x) = 1$ and $\mathbf{q} = q(x) = x^2$. $$\langle \mathbf{p}, \mathbf{q} \rangle = \int_{-1}^{1} p(x)q(x)\,dx$$ (i) $\langle p, q \rangle$ (ii) $d(p, q)$ (iii) $\lVert p \rVert$ (iv) $\lVert q \rVert$	2	1+2+2