JAYPEE UNIVERSITY OF INFORMATION TECHNOLOGY, WAKNAGHAT

TEST -2EXAMINATION- 2025

B.Tech. -III Semester (CSE/IT/CSE-CS/FSSD/AIDS/AIML/UXUI)

COURSE CODE (CREDITS): 25B11MA314 (4) MAX. MARKS: 25

COURSE NAME: Mathematical Foundations for Artificial Intelligence and Data Science

COURSE INSTRUCTORS: RAD, BKP, SST MAX. TIME: 1 Hour 30 Min

Note: (a) All questions are compulsory.

(b) The candidate is allowed to make suitable numeric assumptions wherever required for solving problems

(c) Use of a scientific calculator is allowed.

Q. No.	Question	CO	Marks
Q1	Let S = Span $\{v_1, v_2, v_3\}$ be a subspace of $\mathbb{R}^3$: $$v_1 = \begin{pmatrix}1\\1\\1\end{pmatrix}, \quad v_2 = \begin{pmatrix}1\\2\\0\end{pmatrix}, \quad v_3 = \begin{pmatrix}2\\0\\1\end{pmatrix}.$$ **a)** Use Gram-Schmidt process to construct an orthogonal basis for S. **b)** Normalize vectors obtained to form an orthonormal basis for S.	1	5
Q2	Consider the following 2×2 matrix $A = \begin{bmatrix}1 & 2\\2 & 1\end{bmatrix}$. **a)** Find the singular values of A. **b)** Compute the matrices U, ∑, V such that $A = U\sum V^T$. **c)** Verify your decomposition by reconstructing *A*.	1	4
Q3	The probability that an automobile being filled with gasoline also needs an oil change is 0.25; the probability that it needs a new oil filter is 0.40; and the probability that both the oil and the filter need changing is 0.14. **a)** If the oil has to be changed, what is the probability that a new oil filter is needed? **b)** If a new oil filter is needed, what is the probability that the oil has to be changed?	2	4
Q4	A firm is accustomed to training operators who do certain tasks on a production line. Those operators who attend the training course are known to be able to meet their production quotas 90 % of the time. New operators who do not take the training course only meet their quotas 65 % of the time. Fifty percent of new operators attend the course. Given that a new operator meets her production quota, what is the probability that she attended the program?	2	4

Q5	A discrete random variable X takes the values 1, 2, 3, 4 with corresponding probabilities, respectively. $\frac{1}{10}, \frac{2}{10}, \frac{3}{10}, \frac{4}{10}$ **a)** Find the mean and variance of X. **b)** Define a new random variable $Y=2X+3$. Evaluate the mean and variance of Y.	2	4
Q6	A computer network monitors the packet transmission delay T (in milliseconds) for a data packet sent between two servers. The delay is modeled as a continuous random variable with the following probability density function (PDF): $f(t) = \begin{cases} 3t^2, & 0 < t < 1 \\ 0, & elsewhere \end{cases}$ **a)** Verify that $f(t)$ is a valid probability density function. **b)** Find the cumulative distribution function of the transmission delay.	2	4