JAYPEE UNIVERSITY OF INFORMATION TECHNOLOGY, WAKNAGHAT

Make-up Examination-Nov-2025

COURSE CODE (CREDITS): 24B11CI311 (03) MAX. MARKS: 25

COURSE NAME: COMPUTATIONAL FUNDAMENTALS FOR OPTIMIZATION

COURSE INSTRUCTORS: RBT, VSG MAX. TIME: 1 Hour 30 Minutes

Note: *Note: (a) All questions are compulsory.*

(b) The candidate is allowed to make Suitable numeric assumptions wherever required for solving problems

Q.No	Question	CO	Marks
Q1	a) Suppose that you want to find values for a, b, and c such that the parabola $y = a x^2 + b x + c$ passes through the points (1, 1), (2, 4), and (−1, 1). Find (but do not solve) a system of linear equations whose solutions provide values for a, b, and c. How many solutions would you expect this system of equations to have, and why? b) Find the reduced row echelon form of the augmented matrix for the linear system: $6x_1 + x_2 + 4x_4 = -3$ $-9x_1 + 2x_2 + 3x_3 - 8x_4 = 1$ $7x_1 - 4x_3 + 5x_4 = 2$ Use your result to determine whether the system is consistent and, if so, find its solution.	1	2.5 + 2.5
Q2	In parts (a)–(e) determine whether the statement is true or false, and justify your answer. (a) The transpose of an upper triangular matrix is an upper triangular matrix. (b) All entries of an upper triangular matrix are determined by the entries occurring on and above the main diagonal. (c) The inverse of an invertible lower triangular matrix is an upper triangular matrix. (d) A diagonal matrix is invertible if and only if all of its diagonal entries are positive. (e) If A and B are $n \times n$ matrices such that $A + B$ is symmetric, then A and B are symmetric.	1	1*5 = 5
Q3	a) Let V be an inner product space. Show that if **u** and **v** are	2	1 + 2 + 2= 5

	orthogonal unit vectors in V, then $\lVert \mathbf{u} - \mathbf{v} \rVert = \sqrt{2}$. b) Let P^2 have the inner product $$\langle \mathbf{p}, \mathbf{q} \rangle = \int_{-1}^{1} p(x)q(x)\,dx$$ and let $\mathbf{p} = x$ and $\mathbf{q} = x^2$. Compute the following: $\lVert \mathbf{p} \rVert$, $\lVert \mathbf{q} \rVert$, $\langle \mathbf{p}, \mathbf{q} \rangle$ c) State and prove Cauchy–Schwarz Inequality.		
Q4	a) Write the algorithmic steps for the Gram–Schmidt Process. b) Find a QR-decomposition of the matrix A $$A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}, \; Q = \begin{bmatrix} \frac{1}{\sqrt{5}} & -\frac{2}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \end{bmatrix}$$ **OR** a) Find a singular value decomposition of the matrix $$\begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}.$$ b) Determine whether the following vectors are orthogonal with respect to the Euclidean inner product. u = (- 4 , 6 , -10, 1) , v = (2 , 1, -2, 9)	3	2.5+ 2.5 = 5
Q5	a) Find $E[X]$, where X is the outcome when we roll a fair die. b) Five fair coins are flipped. If the outcomes are assumed independent, find the probability mass function of the number of heads obtained. c) Classify the matrix [3121−13232] as positive definite, negative definite, or indefinite matrix. $$\begin{bmatrix} 3 & 1 & 2 \\ 1 & -1 & 3 \\ 2 & 3 & 2 \end{bmatrix}$$	4	2 + 2 + 1 = 5