Jaypee University of Information Technology, Waknaghat

Makeup Examinations, November 2025

B.Tech - VII Semester (ALL)

Course Code/Credits: 22B1WMA731/3 — Max. Marks: 25
Course Title: Linear Algebra for Data Science & Machine Learning
Course Instructor: RAD — Max. Time: 90 mins

Note: (a) ALL questions are compulsory.

(b) The candidate is allowed to make suitable numeric assumptions wherever required.

Q.No	Question	CO	Marks
Q1	Determine whether each subset is a subgroup of the given group. Justify your answer. (a) $\mathbf{H}_1 = \{x \in \mathbb{Z} : x \text{ is a multiple of } 3\} \subset (\mathbb{Z}, +)$. (b) $\mathbf{H}_2 = \{x \in \mathbb{R} : x > 0\} \subset (\mathbb{R}, \times)$.	CO-1	4
Q2	Let $\mathbf{T} : \mathbb{R}^2 \to \mathbb{R}^3$ be defined by $\mathbf{T}(x) = \mathbf{A}x$, where $\mathbf{A} = \begin{pmatrix} 2 & -1 \\ 4 & 3 \\ -2 & 5 \end{pmatrix}$. Find $x \in \mathbb{R}^2$ such that $\mathbf{T}(x) = \begin{pmatrix} 6 \\ 10 \\ -4 \end{pmatrix}$.	CO-2	4
Q3	Let $\mathbf{U} = \text{span}\{\mathbf{u}_1, \mathbf{u}_2\}$ be a subspace of $\mathbb{R}^4$ generated by $$\mathbf{u}_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 2 \end{pmatrix}, \quad \mathbf{u}_2 = \begin{pmatrix} 0 \\ 1 \\ -1 \\ 1 \end{pmatrix}.$$ (a) Find a basis for $\mathbf{U}$. (b) Determine a basis for the *orthogonal complement* of $\mathbf{U}$.	CO-2	4
Q4	Consider $\mathbf{v} = (2, 1, 0)$ in $\mathbb{R}^3$. (a) Identify the subspace $\mathbf{W}$ defined by $$\begin{aligned} x + y + z &= 0, \\ 2x - y + 3z &= 0. \end{aligned}$$ (b) Find the *orthogonal projection* of $\mathbf{v}$ onto $\mathbf{W}$.	CO-2	4
Q5	Consider the linear transformation $\mathbf{T} : \mathbb{R}^3 \to \mathbb{R}^4$ defined by $\mathbf{T}(x_1, x_2, x_3) = (2x_1 - x_2,\ x_1 + x_3,\ 3x_1 - 2x_2 + x_3,\ 4x_2 - x_3)$. (a) Find the image (range space) of $\mathbf{T}$. (b) Determine the null space of $\mathbf{T}$.	CO-2	4

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Q.No	Question	CO	Marks
Q6	The vectors $\{u_1, u_2, u_3\}$ form a basis for a subspace $\mathbf{W}$ of $\mathbb{R}^4$: $$u_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 1 \end{pmatrix}, \quad u_2 = \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \end{pmatrix}, \quad u_3 = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \end{pmatrix}.$$ (a) Use Gram–Schmidt process to obtain an *orthogonal basis.* (b) Normalize the result to get an *orthonormal basis.*	CO-3	5

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