JAYPEE UNIVERSITY OF INFORMATION TECHNOLOGY, WAKNAGHAT

Make-up Examination-Nov-2025

COURSE CODE(CREDITS):25B11MA314 (4) MAX. MARKS: 25

COURSE NAME: Mathematical Foundations for Artificial Intelligence and Data Science

COURSE INSTRUCTORS: RAD, BKP, **SST** MAX. TIME: 1 Hour 30 Minutes

***Note:****(a) All questions are compulsory.*

*(b) The candidate is allowed to make suitable numeric assumptions wherever required for solving problems.*

*(c) Use of a scientific calculator is allowed.*

| Q. No. | Question | CO | Marks |
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| Q1 | Let $W = \text{Span}\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ be a subspace of $\mathbb{R}^3$:<br>$\mathbf{u}_1 = \begin{pmatrix}1\\0\\1\end{pmatrix}, \quad \mathbf{u}_2 = \begin{pmatrix}1\\1\\0\end{pmatrix}, \quad \mathbf{u}_3 = \begin{pmatrix}0\\1\\1\end{pmatrix}.$<br>a) Apply the Gram–Schmidt process to the set $\{u_1, u_2, u_3\}$ to obtain an orthogonal basis for $W$.<br>b) Normalize the resulting orthogonal vectors to obtain an orthonormal basis for $W$. | 1 | 5 |
| Q2 | Consider the following square matrix:<br>$\mathbf{A} = \begin{bmatrix}3 & 1\\1 & 3\end{bmatrix}$<br>a) Determine the singular values of **A**.<br>b) Find the matrices U,V, $\Sigma$ such that $\mathbf{A} = U\Sigma V^T$.<br>c) Confirm your answer by reconstructing **A** from the obtained matrices. | 1 | 4 |
| Q3 | Consider an industrial process in the textile industry in which strips of a particular type of cloth are being produced. These strips can be defective in two ways, length and nature of texture. It is known from historical information on the process that 10% of strips fail the length test, 5% fail the texture test, and only 0.8% fail both tests. If a strip is selected randomly from the process and a quick measurement identifies it as failing the length test, what is the probability that it is texture defective? | 2 | 4 |
| Q4 | A manufacturing firm employs three analytical plans for the design and development of a particular product. For cost reasons, all three are used at varying times. In fact, plans 1, 2, and 3 are used for 30%, 20%, and 50% of the products, respectively. The defect rate is different for the three procedures as follows:<br>$P(D\|P1) = 0.01, P(D\|P2) = 0.03, P(D\|P3) = 0.02,$ | 2 | 4 |





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| | where $P(D\|Pj)$ is the probability of a defective product, given plan $j$. If a random product was observed and found to be defective, which plan was most likely used and thus responsible? | | |
| Q5 | A continuous random variable $X$ has the probability density function (pdf) given by<br>$$f(x) = \begin{cases} kx^2, & 0 \le x \le 2, \\ 0, & \text{otherwise.} \end{cases}$$<br>a) Find the value of $k$.<br>b) Define a new random variable $Y = 3X - 2$. Evaluate the mean and variance of $Y$. | 2 | 4 |
| Q6 | The lifetime $T$ (in years) of a certain type of electronic component is modeled as a continuous random variable with the following probability density function (PDF):<br>$$f(t) = \begin{cases} 2t, & 0 < t < 1, \\ 0, & \text{elsewhere.} \end{cases}$$<br>a) Verify that $f(t)$ is a valid probability density function.<br>b) Find the cumulative distribution function (CDF) $F(t)$ of the component lifetime.<br>c) Determine $P(T < 0.5)$ and $P(0.25 < T < 0.75)$. | 2 | 4 |