

**JAYPEE UNIVERSITY OF INFORMATION TECHNOLOGY, WAKNAGHAT**

**Make-up Examination-Nov-2025**

**COURSE CODE (CREDITS): 22M11MA111 (3)**

**MAX. MARKS: 25**

**COURSE NAME: Mathematical Foundations for Data Science**

**COURSE INSTRUCTORS: RVS**

**MAX. TIME: 1 Hour 30 Min**

**Note: Note: (a) All questions are compulsory.**

**(b) The candidate is allowed to make Suitable numeric assumptions wherever required for solving problem.**

Q.No	Question	CO	Marks
Q1	(a) Explain why data is essential for Machine Learning. How does the nature of data influence the choice between supervised, unsupervised, and reinforcement learning? Give one example for each. (b) Define a field and its main properties. Give one numerical example showing closure and existence of inverse elements.	CO1	3+2
Q2	(a) Define a vector space and list the axioms that must be satisfied for a set to qualify as a vector space. (b) Consider the set $S = \{(x, y, z) \in \mathbb{R}^3 \mid x + y + z = 0\}.$ Show that $S$ is a subspace of $\mathbb{R}^3$ . Find a basis for $S$ and determine its dimension.	CO2	2+3
Q3	(a) Define linear independence. Explain how it helps in identifying redundant features in datasets. (b) Determine whether the following vectors are linearly independent: Justify your answer. $a = (1, 2, 4), b = (2, 5, 7), c = (1, 0, 1)$	CO2	2+3
Q4	(a) Explain the concept of <i>diagonalization</i> of a matrix. Under what condition is a matrix said to be diagonalizable? (2 Marks) (b) Find the eigenvalues and eigenvectors of $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ and determine if $A$ is diagonalizable. If yes, find the diagonal matrix $D$ and the corresponding matrix $P$ such that $A = PDP^{-1}$ .	CO3	2+3
Q5	a) Machine Learning is a subset of Data Science. (T/F) b) In a field, every non-zero element has a multiplicative _____. c) The process of expressing a vector as a combination of basis vectors is called _____. d) The zero vector is always linearly independent. (T/F) e) The number of vectors in a basis is called the _____ of the vector space. f) A set of vectors is linearly dependent if at least one vector can be written as a _____ of others. g) A matrix is diagonalizable if it has enough linearly _____ eigenvectors. h) The determinant of a diagonal matrix is the _____ of its diagonal entries. i) In a linear transformation $T$ , $T(au + bv) = aT(u) + bT(v)$ . (T/F) j) Eigenvectors corresponding to distinct eigenvalues are always _____.	CO1-3	5