JAYPEE UNIVERSITY OF INFORMATION TECHNOLOGY, WAKNAGHAT

Comprehensive Examination - 2025

Ph.D. (CSE//ECE/CE/BT/BI/PMS/MATHS/HSS)

COURSE CODE (CREDITS): MAX. MARKS: 100

COURSE NAME: Comprehensive Paper

COURSE INSTRUCTORS: RKB MAX. TIME: 3 Hours

Note: *(a) All questions are compulsory.*

(b) The candidate is allowed to make Suitable numeric assumptions wherever required for solving problems

Sec A

Q.No	Question	Marks
Q1	Explain the comparative aspects for membership (fuzzy) function and characteristic (crisp) function with the help of numerical examples.	7
Q2	How do you define an intuitionistic fuzzy information and detail about its extension to picture fuzzy information? Provide a real-life example where picture fuzzy information is applicable. Use suitable diagram to explain the same.	7
Q3	Let A = {⟨x_1, 0.6, 0.2⟩, ⟨x_2, 0.8, 0.1⟩, ⟨x_3, 0.5, 0.3⟩} and B = {⟨x_1, 0.7, 0.1⟩, ⟨x_2, 0.6, 0.2⟩, ⟨x_3, 0.4, 0.4⟩} be two Intuitionistic Fuzzy Sets (IFS) defined on the universe X = {x_1, x_2, x_3}, where each element is represented as ⟨x_i, $\mu(x_i)$, $v(x_i)$⟩, with $\mu(x_i)$ as the degree of membership and $v(x_i)$ as the degree of non-membership. Compute the union A ∪ B, intersection A ∩ B and complement A′. Also, find the Hamming distance and Euclidean distance between them.	8
Q4	Define a neutrosophic fuzzy set and explain how it has been extended from IFS. Illustrate the process with the help of example capturing indeterminacy more effectively than IFS.	7
Q5	What is the purpose of normalized Euclidean distance measure in case of IFS? Explain the process of normalization with the help of suitable numerical example.	5

Sec B

Q.No	Question	Marks
Q6	Prove or disprove that $f(x) = \frac{1}{1+x^2}$ is uniformly continuous on $\mathbb{R}$.	9
Q7	Evaluate $\oint_\gamma \frac{3z^3+4z^2-5z+1}{(z^3-z)(z-2i)}\,dz$ where contour $\gamma: \lvert z\rvert = 3$ is taken in positive sense.	9
Q8	Define the inner product on $\mathbb{R}^2$, and prove or disprove that for $\boldsymbol{v}, \boldsymbol{w} \in \mathbb{R}^2$ the product given by $\langle \boldsymbol{v}, \boldsymbol{w} \rangle = v_1w_1 - v_1w_2 - v_2w_1 + 4v_2w_2$ is an inner product. Take $\boldsymbol{v} = (v_1, v_2), \boldsymbol{w} = (w_1, w_2)$.	9
Q9	Consider a sequence (a_n), where $a_n = \frac{1}{2} + \frac{1}{2}\sin\left(\frac{n\pi}{3}\right)$. Compute $\liminf a_n$, and $\limsup a_n$.	6

Sec C

Q. No.	Question	Marks
Q1	Consider the following system of equations: $$\begin{aligned} x+2y+z &= 3 \\ 2x+5y+3z &= 8 \\ x+y+z &= 2 \end{aligned}$$ (a) Reduce the augmented matrix to *reduced row-echelon form* using elementary row operations. (b) Determine whether the system is consistent.	6
Q2	Let $T:\mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation defined by $T(x,\ y,\ z) = \begin{pmatrix} x+y \\ y+z \\ x+z \end{pmatrix}$. (a) Find the matrix representation of T with respect to the standard basis. (b) Determine the *rank* and *nullity* of T. (c) Find a *basis* for the range and *kernel* of T.	7
Q3	Consider the following 3×3 diagonal matrix: $$A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}.$$ (a) Find the *eigenvalues* and *eigenvectors* of the matrix A. (b) Determine whether A is *diagonalizable*.	7
Q4	Answer the following question. (a) State and prove the *Cayley–Hamilton Theorem*. (b) Using it, find B^4 for $B = \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$.	7
Q5	Consider the following 3×3 matrix: $$C = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}.$$ (a) Find the *characteristic* and *minimal polynomials* of C. (b) Find the *Jordan canonical form* of C.	6

Sec C

Q.No	Question	Marks
Q7	(a) Explain the Born-Oppenheimer approximation and why is it called adiabatic approximation? (b) Write down the total Hamiltonian of an N_e electron and NA atoms solid and explain each term in the Hamiltonian. (c) Use Born-Oppenheimer approximation to the total Hamiltonian and write the Hamiltonian for electrons only.	1+1 2 2
Q8	(a) What was the failure of Born-Oppenheimer approximation to lead to Hartree approximation? (b) What is Hartree approximation? Explain very well. (c) Describe Hartree-Fock approximation.	3 2 3
Q9	(a) Prove that the Slater determinant supports Pauli exclusion principle. (b) Write down the integral form of exchange integral and Coulomb integral obtained from Hartree-Fock theory.	3 3
Q10	(a) Write down the Hohenberg-Kohn theorem and prove them. (b) Define basis function and how can a total wave function of a molecule be expressed in terms of basis functions?	6 2+5

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