Jaypee University of Information Technology, Waknaghat

TEST-3 Examination - December 2025

B.Tech - VII Semester (ALL)

Course Code/Credits: 22B1WMA731/3 Max. Marks: 35
Course Title: Linear Algebra for Data Science & Machine Learning
Course Instructor: RAD Max. Time: 2 Hours

Note: (a) ALL questions are compulsory.

(b) Scientific calculators are allowed.

(c) The candidate is allowed to make suitable numeric assumptions wherever required.

Q.No	Question	CO	Marks
Q1	Consider the following vectors: $$\mathbf{v}_1 = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \quad \mathbf{v}_2 = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}, \quad \mathbf{v}_3 = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}.$$ (a) Show that $\mathbf{w} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ is a linear combination of $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$. (b) Is $\mathbf{w} = \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$ a linear combination of $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$? Justify.	CO-1	5
Q2	Let $\mathbf{W}$ be the subspace of $\mathbb{R}^3$ spanned by the following vectors: $$w_1 = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}, \qquad w_2 = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}.$$ (a) Find an orthogonal basis for $\mathbf{W}$ using Gram–Schmidt process. (b) Determine the *orthogonal complement* $\mathbf{W}^{\perp}$ of subspace $\mathbf{W}$.	CO-2	5
Q3	Consider the *singular value decomposition* of following 3×2 matrix: $$\mathbf{A} = \begin{pmatrix} 4 & 0 \\ 3 & 1 \\ 0 & 2 \end{pmatrix} = \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^{\mathbf{T}},$$ where $$\mathbf{U} = \begin{pmatrix} \frac{4}{5} & 0 & \frac{3}{5} \\ \frac{3}{5} & 0 & -\frac{4}{5} \\ 0 & 1 & 0 \end{pmatrix}, \quad \boldsymbol{\Sigma} = \begin{pmatrix} 5 & 0 \\ 0 & 2 \\ 0 & 0 \end{pmatrix}, \quad \mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$ (a) Compute the pseudo-inverse $\mathbf{A}^{\dagger}$ of the rectangular matrix $\mathbf{A}$. (b) Verify the identity $\mathbf{A}^{\dagger}\mathbf{A} = \mathbf{V}\mathbf{V}^{\mathbf{T}}$.	CO-2	5

<table>
<tr><th>Q.No</th><th>Question</th><th>CO</th><th>Marks</th></tr>
<tr><td>Q4</td><td>Consider the following inconsistent system of linear equations:
$$\begin{aligned} 2x_1 + 3x_2 &= 5 \\ 4x_1 - x_2 &= 1 \\ 6x_1 + 2x_2 &= 4 \end{aligned}$$
(a) Find the least-squares solution $\hat{\mathbf{x}}$ to the given system $\mathbf{Ax} = \mathbf{b}$.
(b) Find the least-squares error $||\mathbf{b} - \mathbf{A}\hat{\mathbf{x}}||$.</td><td>CO-3</td><td>5</td></tr>
<tr><td>Q5</td><td>A tech company tracks the weekly usage time, in hours, of two software tools, Tool A and Tool B, by two of its employees:
<table><tr><th>Features</th><th>Employee 1</th><th>Employee 2</th></tr><tr><td>Tool A</td><td>6</td><td>9</td></tr><tr><td>Tool B</td><td>12</td><td>7</td></tr></table>
The company wants to reduce this dataset to a lower-dimensional representation while retaining most of the variance. Perform principal component analysis (PCA) on this dataset to answer:
(a) Compute the covariance matrix $\mathbf{S}$ of the centered data $\mathbf{X}$.
(b) Determine the eigenvalues and eigenvectors of $\mathbf{S}$.
(c) Form the orthogonal matrix $\mathbf{P}$ using the eigenvectors of $\mathbf{S}$.
(d) Project dataset $\mathbf{X}$ onto the first principal component (PC1).
(e) What proportion of the total variance is captured by PC1?</td><td>CO-3</td><td>8</td></tr>
<tr><td>Q6</td><td>An AIML research team wants to predict the accuracy of a classification model based on the number of training epochs. The team collects experimental data showing how accuracy (%) improves as training progresses. The training dataset is given below:
<table><tr><th>Epochs (x)</th><th>Accuracy (y)</th></tr><tr><td>5</td><td>58</td></tr><tr><td>10</td><td>67</td></tr><tr><td>15</td><td>74</td></tr></table>
The objective is to learn the relationship between the number of epochs x and the model accuracy y using linear regression with gradient descent. The prediction model is $\hat{y} = \theta_0 + \theta_1 x$. Assume the learning rate as $\alpha = 0.01$.
(a) Define the cost function for this regression task.
(b) Perform first iteration of gradient descent with $\theta_0 = 0$, $\theta_1 = 0$.</td><td>CO-4</td><td>7</td></tr>
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