JAYPEE UNIVERSITY OF INFORMATION TECHNOLOGY, WAKNAGHAT
TEST-3 EXAMINATION - 2025
B.Tech.-III Semester (CSE/IT/CSECS/FSSD/AIDS/AIML/UXUI)
COURSE CODE (CREDITS): 25B11MA314 (4) MAX. MARKS: 35
COURSE NAME: Mathematical Foundations for Artificial Intelligence and Data Science
COURSE INSTRUCTORS: RAD, BKP, SST MAX. TIME: 2 Hours

Note: *(a) All questions are compulsory.*

(b) The candidate is allowed to make suitable numeric assumptions wherever required for solving problems.

(c) Use of a scientific calculator is allowed.

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<table>
<tr><th>Q. No.</th><th>Question</th><th>CO</th><th>Marks</th></tr>
<tr><td>Q1</td><td>Consider the vectors in $\mathbb{R}^3$; $v_1 = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}, v_2 = \begin{pmatrix} 3 \\ -1 \\ 4 \end{pmatrix}$.
Let $\boldsymbol{W}$ =span$\{v_1, v_2\}$ be the subspace of $\mathbb{R}^3$ generated by v_1, v_2.
a) Determine whether v_1, and $v_2$are linearly independent.
b) Find a basis for the subspace $\boldsymbol{W}$.</td><td>1</td><td>5</td></tr>
<tr><td>Q2</td><td>a) Consider the joint density function of X and Y be
$$f_{XY}(x,y) = \begin{cases} \frac{1}{9}xy, & 0 < x < 2, 0 < y < 3 \\ 0, & otherwise \end{cases}$$
Determine the marginal probability density function $f_X(x)$.
b) In a large university network, the probability that a randomly selected computer is infected is 0.06. A security team randomly scans 50 computers for infections What is the probability that at least 2 computers are infected?</td><td>3</td><td>5</td></tr>
<tr><td>Q3</td><td>The response time $\boldsymbol{X}$ in milliseconds (ms) of a cloud server handling user requests is normally distributed with a mean of 120 ms and a standard deviation of 20 ms.
a) Determine $P(\boldsymbol{X} > 150)$.
b) Find the response time that is exceeded by 5% of the requests.</td><td>3</td><td>5</td></tr>
<tr><td>Q4</td><td>A smart city monitoring system records traffic density (vehicles per minute) and the corresponding average waiting time (seconds) at a traffic signal) for 8 peak-hour observations:
<table>
<tr><td>Observation</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td></tr>
<tr><td>Traffic Density (vehicles/min)</td><td>20</td><td>25</td><td>30</td><td>40</td><td>45</td><td>50</td><td>55</td><td>60</td></tr>
<tr><td>Waiting Time (seconds)</td><td>25</td><td>32</td><td>35</td><td>48</td><td>55</td><td>60</td><td>65</td><td>72</td></tr>
</table>
a) Compute the Karl Pearson's correlation coefficient between traffic density and waiting time.</td><td>2</td><td>5</td></tr>
</table>

<table>
<tr><td></td><td>b) Interpret whether increasing vehicle density significantly affects the waiting time.</td><td></td><td></td></tr>
<tr><td>Q5</td><td>A mechanical engineer wants to estimate the fuel consumption of a diesel engine based on its load (kg). The following experimental data was recorded:
<table>
<tr><td>Load X (kg)</td><td>5</td><td>10</td><td>15</td><td>18</td><td>22</td><td>25</td></tr>
<tr><td>Fuel Consumption Y (lit/hr.)</td><td>1.2</td><td>2.3</td><td>3.1</td><td>3.6</td><td>4.2</td><td>4.8</td></tr>
</table>
a) Fit the best-fit regression line using least square estimation.
b) Estimate the expected fuel consumption when the load on the engine is 20 kg.</td><td>2</td><td>5</td></tr>
<tr><td>Q6</td><td>Solve the following linear programming problem graphically:
$$Max\ Z = 8000\,x_1 + 7000\,x_2$$
$$s.t.$$
$$3x_1 + x_2 \le 66,$$
$$x_1 + x_2 \le 45,$$
$$x_2 \le 40,$$
$$x_1, x_2 \ge 0$$
<i>(No graph paper will be provided to solve this question. Please draw on the sheet of answer script.)</i></td><td>4</td><td>4</td></tr>
<tr><td>Q7</td><td>Solve the following linear programming problem by simplex method:
$$Max\ Z = 3x_1 + 2x_2$$
$$s.t.$$
$$2x_1 + x_2 \le 10,$$
$$x_1 + 3x_2 \le 6,$$
$$x_1, x_2 \ge 0$$</td><td>4</td><td>6</td></tr>
</table>

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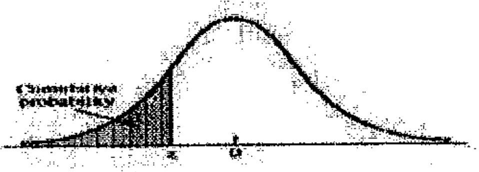
(Standard) Normal Probability Table:



z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
-1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
-1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
-0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
-0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]