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Jaypee University of Information Technology, Waknaghat

TEST-3 Examination - May 2026

B.Tech - VII Semester (ALL)

Course Code/Credits: 22B1WMA731/3 — Max. Marks: 35
Course Title: Linear Algebra for Data Science & Machine Learning
Course Instructor: RAD — Max. Time: 2 Hours

Note: (a) ALL questions are compulsory.
(b) Scientific calculators are allowed.
(c) The candidate is allowed to make suitable numeric assumptions wherever required.

Q.No	Question	CO	Marks
Q1	Consider the following sets of vectors: $$\mathbf{E}_1 = \left\{ \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 4 \\ 3 \end{pmatrix}, \begin{pmatrix} 6 \\ 5 \end{pmatrix} \right\}$$ $$\mathbf{E}_2 = \left\{ \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}, \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} \right\}.$$ (a) Are the sets of vectors linearly independent? (b) Describe the span of each set.	CO-1	5
Q2	Determine a basis from the following vectors for $\mathbb{R}^3$: $$\mathbf{v}_1 = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \mathbf{v}_2 = \begin{pmatrix} 2 \\ 3 \\ 5 \end{pmatrix} \quad \mathbf{v}_3 = \begin{pmatrix} 2 \\ 6 \\ 8 \end{pmatrix} \quad \mathbf{v}_4 = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}.$$	CO-1	5
Q3	Let $\mathbf{W}$ be the subspace spanned by the given vectors: $$\mathbf{w}_1 = \begin{bmatrix} 0 \\ -6 \\ 3 \\ -3 \end{bmatrix} \quad \text{and} \quad \mathbf{w}_2 = \begin{bmatrix} 0 \\ 0 \\ 3 \\ 3 \end{bmatrix}.$$ (a) Find the `orthogonal complement` of $\mathbf{W}$. (b) What is the dimension of $\mathbf{W}^{\perp}$?	CO-2	5
Q4	Consider the following 4×2 matrix: $$\mathbf{A} = \begin{bmatrix} 1 & 1 \\ -1 & 1 \\ 1 & 1 \\ -1 & 1 \end{bmatrix}$$ Apply Gram-Schmidt process to the column vectors of $\mathbf{A}$.	CO-3	6

<table>
<tr><th>Q.No</th><th>Question</th><th>CO</th><th>Marks</th></tr>
<tr><td>Q5</td><td>Let B be the given 2×3 matrix:

$$\begin{bmatrix} 1 & -1 & 1 \\ 1 & -1 & 1 \end{bmatrix}$$

(a) Determine the singular values of B.
(b) Compute singular value decomposition (SVD) of B.</td><td>CO-3</td><td>6</td></tr>
<tr><td>Q6</td><td>A tech company tracks the weekly usage time, in hours, of two software tools, Tool A and Tool B, by two of its employees:
<table>
<tr><th>Features</th><th>Employee 1</th><th>Employee 2</th></tr>
<tr><td>Tool A</td><td>6</td><td>9</td></tr>
<tr><td>Tool B</td><td>12</td><td>7</td></tr>
</table>
The company wants to reduce this dataset to a lower-dimensional representation while retaining most of the variance. Perform principal component analysis (PCA) on this dataset to answer:
(a) Compute the covariance matrix S of the centered data X.
(b) Determine the eigenvalues and eigenvectors of S.
(c) Form the orthogonal matrix P using the eigenvectors of S.</td><td>CO-3</td><td>8</td></tr>
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