

JAYPEE UNIVERSITY OF INFORMATION TECHNOLOGY, WAKNAGHAT

TEST -3 EXAMINATION- 2026

B.Tech.- III Semester (CSE/IT/CSECS/FSSD/AIDS/AIML/UXUI)

COURSE CODE (CREDITS):25B11MA314 (4)

MAX. MARKS: 35

COURSE NAME: Mathematical Foundations for Artificial Intelligence and Data Science

COURSE INSTRUCTORS: RKB

MAX. TIME: 2 Hours

Note: (a) All questions are compulsory.

(b) The candidate is allowed to make suitable numeric assumptions wherever required for solving problems. (c) Use of a scientific calculator is allowed.

Q.No.	Question	CO	Marks
Q1	If $T : R^3 \rightarrow R^3$ be the linear transformation defined by $T(x, y, z) = (x + 2y - z, y + z, x + y - 2z)$. Find a basis and the dimension of: (a) the range space of T , (b) the null space (kernel) of T .	1	5
Q2	A defect detection system has a probability $p = 0.2$ of incorrectly classifying an item. (a) Find the probability that no errors occur in 8 independent classifications. (b) Find the probability that more than 2 errors occur in 15 classifications. (c) Find the minimum number of classifications required so that the expected number of errors is at least 4. (d) Find the smallest number of classifications such that the probability of at least one error exceeds 0.90	2	5
Q3	Solve the following linear programming problem by simplex method: $\begin{aligned} \text{Max } Z &= 7x_1 + 5x_2 \\ \text{s.t.} \\ x_1 + 2x_2 &\leq 6, \\ 4x_1 + 3x_2 &\leq 12, \\ x_1, x_2 &\geq 0 \end{aligned}$	4	5
Q4	Determine whether the following sets are convex or not. Justify your answer: a) Show that in R^3 , the closed ball $S = \{(x_1, x_2, x_3) : x_{21} + x_{22} + x_{23} \leq 1\}$ is a convex set. b) Show that the set $C = \{(x_1, x_2) \in R^2 : 2x_1 + 3x_2 = 7\}$ is a convex set.	4	5

Q5	Find the dual of the following primal problem: $\text{Max } Z = 3x_1 - 2x_2 + 4x_3$ s. t. $3x_1 + 5x_2 + 4x_3 \leq 7,$ $6x_1 + x_2 + 3x_3 \leq 4,$ $7x_1 - 2x_2 - x_3 \leq 10,$ $x_1 - 2x_2 + 5x_3 \leq 3,$ $4x_1 + 7x_2 - 2x_3 \leq 2$ $x_1, x_2, x_3 \geq 0$	4	5
Q6	Write the Kuhn-Tucker conditions for the following minimization problem: $\text{Min } f(x) = x_1^2 + x_2^2 + x_3^2$ s. t. $x_1 + x_2 \leq 5,$ $2x_1 + x_3 \leq 4,$ $x_1 \geq 1,$ $x_2 \geq 2,$ $x_3 \geq 0$	4	5
Q7	Verify which of the following are vector spaces or not: (a) $V = \{x = (x_1, x_2) \in \mathbb{R}^2 \mid x_1 = 3x_2\}$ (b) $V = \{(x_1, x_2) \in \mathbb{R}^2 \mid x_1 > 0, x_2 > 0\}$ (c) $V = \{(x_1, x_2, x_3) \in \mathbb{R}^3 \mid x_1 + 2x_2 + 3x_3 = 5\}$ (d) $M_{2 \times 2} = \left\{ \begin{bmatrix} a & 0 \\ 0 & d \end{bmatrix}; a, b \in \mathbb{R} \right\}$ under usual matrix operations.	1	5