

Note: (a) All questions are compulsory.
(b) The candidate is allowed to make suitable numeric assumptions wherever required for solving problems
(c) Use of calculator is not allowed

Q.No	Question	CO	Marks
Q1	a) Determine whether the function $f(x, y) = \begin{cases} \frac{x^3 y}{x^6 + y^2}, & (x, y) \neq (0, 0), \\ 0, & (x, y) = (0, 0), \end{cases}$ is continuous at (0,0). Justify your answer. b) If $u = \sin^{-1} \left(\frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}} \right)$, show that $\frac{\partial u}{\partial x} = -\frac{y}{x} \frac{\partial u}{\partial y}$	CO-1	4
Q2	a) The temperature at a point inside a material depends on three physical quantities -pressure p, density ρ, and velocity v according to $T(p, \rho, v) = p^2 + 2\rho v + v^2.$ These quantities depend on two spatial coordinates x and y as $p = x + y, \quad \rho = x^2 + y, \quad v = xy.$ Using the chain rule of partial differentiation, determine $\frac{\partial T}{\partial x}$ and $\frac{\partial T}{\partial y}$ at the point $(x, y) = (1, 2)$. b) The temperature distribution is $T(x, y) = \log_e(xy) + x^2 y^2$. Verify that $\frac{\partial^2 T}{\partial x \partial y} = \frac{\partial^2 T}{\partial y \partial x}$	CO-1	4
Q3	Let $f(x, y) = x^2 y + \sin(xy)$. Find the second-degree Taylor's polynomial of f expanded in powers of $(x - 1)$ and y . Also, use it to estimate $f(1.1, 0.1)$.	CO-1	3.5
Q4	Using constrained optimization (or Lagrange multipliers), determine the optimal parameter configuration (x, y, z) in an AI/ML model for $f(x, y, z) = x^2 + 2y - z^2$; where x, y, z represent three model parameters satisfying two linear constraints $2x - y = 0$ and $y + z = 0$.	CO-1	3.5