JAYPEE UNIVERSITY OF INFORMATION TECHNOLOGY, WAKNAGHAT

TEST -2 EXAMINATION- October 2018

B.Tech (CSE&IT) VII Semester

Dr. R. Bhatt

COURSE CODE: 10B1WCI733

MAX. MARKS: 25

COURSE NAME: Graph Algorithms and Applications

COURSE CREDITS: 3

MAX. TIME: 1Hr 30 Min

***Note:** All questions are compulsory.*

1. [2.5 + 2.5 Marks] [CO 3]

a. State and prove Hall's theorem.

b. The people in a club are planning their summer vacations. Trips $t_1, \ldots t_n$ are available, but trip t_i has capacity c_i. Each person likes some of the trips and will travel on atmost one. In terms of which people like which trips, derive a necessary and sufficient condition for being able to fill all trips (to capacity) with people who like them.

2. [2.5 + 2.5 Marks] [CO 3]

a. Prove or disprove: Every maximal matching in a graph G has at least $\alpha'(G) / 2$ edges ($\alpha'(G)$ is the maximum size of matching).

b. Show how to use the Hungarian Algorithm to test for the existence of a perfect matching in a bipartite graph.

3. [2.5 + 2.5 Marks] [CO 2]

a. Prove or disprove: For a given network, the value of a maximum flow is equal to the capacity of a minimum cut.

b. For the network shown in Figure 1, the arc from vertex v to vertex w has flow capacity 1, while the other arcs have capacity M, which could be made arbitrarily large. If the choice of augmenting flow path at each iteration were to alternate between the directed path (s, v, w, t) and the quasi-path (s, w, v, t). Compute the number of iterations to obtain the maximum flow.

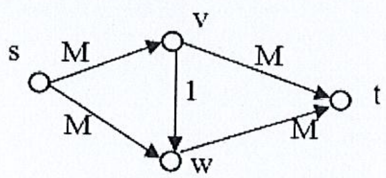

Figure 1

4. [2. 5 + 2.5 Marks] [CO 3]

a. Prove or disprove: If $G = F \cup H$, then $\chi(G) \leq \chi(F) + \chi(H)$. ($\chi(G)$ is the chromatic number of graph G)

b. Provide two examples of applications which can be modeled as vertex-coloring problems.

5. [2.5 + 2.5 Marks] [CO 2]

a. Suppose you are given a weighted graph G = (V, E) with a distinguished vertex s and where all edge weights are positive and distinct. Is it possible for a tree of shortest paths from s and a minimum spanning tree in G to not share any edges? If so, give an example. If not, give a reason.

b. Let G be a graph with more than k vertices that is not a complete graph. Prove that if G is not k-connected, then G has a separating set of size k - 1.