Dr Neelkanth

Note: All questions are compulsory. Carrying of mobile phone during examinations will be treated as case of unfair means. Marks are indicated against each question in square brackets.

1. Find the power series solution of $y'' + 3xy = 0$ about $x = 0$.   [5](CO-2)

2. Express $x^4 + 3x^3 + 2x^2 + 5x + 1$ in terms of Chebyshev polynomial $T_n(x)$.   [2](CO-3)

3. Evaluate $\int_{-1}^{1} P_n^2(x)\,dx$ and hence show that $\int_{-1}^{1} P_4^2(x)\,dx = \dfrac{2}{9}$.   [5](CO-3)

4. Prove that $nP_n(x) = xP_n'(x) - P_{n-1}'(x)$.   [3](CO-3)

5. Find a Fourier series expansion of $f(x) = \pi - x$ for $0 < x < 2\pi$.   [3](CO-4)

6. Find the temperature distribution $u(x,t)$ for a laterally insulated bar of length 80 cm, if the initial temperature of the bar is $100\sin\left(\dfrac{\pi x}{80}\right)°C$ and ends are kept at $0°C$, governed by

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}.$$

     [7](CO-4)