

JAYPEE UNIVERSITY OF INFORMATION TECHNOLOGY, WAKNAGHAT

TEST -3 EXAMINATION- Dec. 2017

Ph.D. Mathematics: I Semester

COURSE CODE: 17P1WMA111

MAX. MARKS: 35

COURSE NAME: DIFFERENTIAL GEOMETRY

COURSE CREDITS: 3

MAX. TIME: 2 Hrs

Note: All questions are compulsory. Carrying of mobile phone during examinations will be treated as case of unfair means.

1. Let $V_1 = U_1 - xU_3$, $V_2 = U_2$ and $V_3 = xU_1 + U_3$. Prove that the vectors $V_1(p), V_2(p), V_3(p)$ are linearly independent at each point $p \in R^3$. [5 Marks]
2. For any function f , show that the vector fields $E_1 = (\sin f U_1 + U_2 - \cos f U_3)/\sqrt{2}$, $E_2 = (\sin f U_1 - U_2 - \cos f U_3)/\sqrt{2}$, $E_3 = (\cos f U_1 + \sin f U_3)$; form a frame field, and find its connection forms. [6 Marks]
3. Define a differentiable manifold and explain it by an example. [6 Marks]
4. Define the normal curvature $K_n(\mathbf{u})$ in the \mathbf{u} direction. Moreover by means of a rough sketch interpret the geometric meaning of $K_n(\mathbf{u}) > 0$ and $K_n(\mathbf{u}) < 0$. [6 Marks]
5. Define the Gaussian curvature K at a point of a surface and using this classify a point p of a surface S as elliptic, hyperbolic, parabolic or planar. [6 Marks]
6. Define the following: [6 Marks]
 - (a) Connected surface
 - (b) Compact Surface
 - (c) Orientable surface
