Dr Rakesh Bajaj

COURSE CODE: 10B11MA211   MAX. MARKS: 50
**COURSE NAME: DISCRETE MATHEMATICS**
COURSE CREDITS: 4   MAX. TIME: 2 HRS

Note: All questions are compulsory. Carrying of mobile phone during examinations will be treated as case of unfair means. Use of Calculator is not allowed. Attempt the questions section wise in a sequence. Questions 1 to 8 are of 5 marks each and 9th question is of 10 marks.

1. Using generating function, solve the recurrence relation $a_k = 7a_{k-1} - 10a_{k-2};\ \ k \geq 2,$ with initial conditions $a_0 = 1$ and $a_1 = 8$.

2. Using mathematical induction, show that $1 + \dfrac{1}{4} + \dfrac{1}{9} + ... + \dfrac{1}{n^2} \leq 2 - \dfrac{1}{n};\ \ \forall n \in set\ of\ natural\ numbers$.

3. Define a r-regular graph with an example. Prove that if a graph of order (number of vertices) $3n,\ (n \geq 1)$ has equal number of vertices of degrees $n - 1,\ n$ and $n + 1$, then n is even.

4. Using adjacency matrix, find the path matrix of the following directed graph:
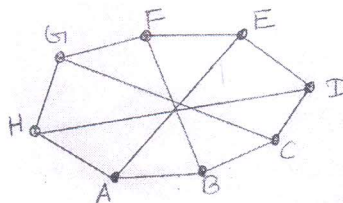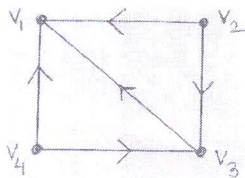
5. State Kuratowski's theorem. What are Kuratowski's graphs? Using theorem, investigate the planarity of the above undirected graph.

6. Define a spanning tree. A certain tree T with 21 vertices has only vertices of degree 1, 3, 5 and 6. If T has exactly 15 end vertices and 1 vertex of degree 6, how many vertices of T have degree 5?

7. Define a bounded, complemented and distributive lattice. Also give examples of non-distributive lattices.

8. What is the Kleene closure of a language? Find the language of the following grammar:

   G = (V, T, S, P), where V = {S, W, a, b, c}, T = {a, b, c}, S = {S}, P = {S → aW, W → bbW, W → c}.

9. Give an example if possible:

   (a) Hamiltonian Graph with 5 vertices.
   (b) Eulerian graph but not Hamiltonian.
   (c) Two isomorphic graphs
   (d) Two homeomorphic graphs.
   (e) A Poset
   (f) Bounded Lattice
   (g) Group but not an abelian group
   (h) Field with two elements.
   (i) A Group with 4 elements.
   (j) Complete bipartite graph which is a tree.

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