

JAYPEE UNIVERSITY OF INFORMATION TECHNOLOGY, WAKNAGHAT

TEST-3 EXAMINATION- JUNE -2016

Ph.D

COURSE CODE: 13M1WEC132

MAX. MARKS: 35

COURSE NAME: Mathematical Techniques for Engineering

COURSE CREDITS: 03

MAX. TIME: 2 HRS

*Note: All questions are compulsory. Carrying of mobile phone during examinations will be treated as case of unfair means. Each question carrier five marks.*

1. Give the properties of probability density function and cumulative distribution function. The joint probability density function of the random variables X and Y is given by  $f_{XY} = Ce^{-(ax+by)}u(x)u(y)$  where a, b and C are constants. Find the value of C. Find  $P(X \leq 1, Y \leq 1)$ . Specify whether the random variables are independent or not.  $u(t)$  is a unit step function of  $t$ .
2. Find the values of b and c for which the matrix  $A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 3 & b \\ 0 & b & c \end{bmatrix}$  has  $[1 \ 0 \ 1]^T$  as an eigen vector. For these values of b and c calculate the Eigen values and the matrix A.
3. A periodic function  $f(t)$ , of period  $2\pi$ , is defined within the period  $-\pi < t < \pi$  by  $f(t) = \begin{cases} 0 & (-\pi < t < 0) \\ 1 & (0 < t < \pi) \end{cases}$ . Using the Fourier series coefficients of  $f(t)$ , together with Parseval's theorem, show that  $\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} = \pi^2/8$
4. Solve the difference equation  $y_{k+2} + 2y_k = 0, k \geq 0$  given that  $y_0 = 1$  and  $y_1 = \sqrt{2}$ .
5. Obtain the inverse Laplace transform of  $\frac{s-3}{(s-1)^2(s-2)}$
6. What do you mean by Ergodic random process? Obtain the relationship between the power spectral density and auto correlation function.
7. Define the following briefly.
  - a. Conditional probability.
  - b. Orthogonality and orthonormality.
  - c. Linear dependence and linear independence.
  - d. Region of convergence for Laplace transform.
  - e. Conditions for the existence of Fourier series expansion.