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JAYPEE UNIVERSITY OF INFORMATION TECHNOLOGY, WAKNAGHAT
FINAL EXAMINATION
SUMMER SEMESTER JUNE-JULY 2016

COURSE NAME: MATHEMATICS-II MAXM. MARKS : 50
COURSE CODE : 10B11MA201
COURSE CREDITS : 04 MAXM. TIME: 2 HOURS

NOTE: There are ten questions in all. Attempt all questions. Use of any calculator is not permitted. All questions carry equal marks.

1. a. Show that $f(z) = \bar{z}$ is continuous but not differentiable at any point.
   b. If $f(z) = x^2 + iy^2$, does $f'(z)$ exist at any point?

2. Determine the analytic function $f(z) = u + iv$, where $u = x^3 - 3xy^2 - 2x$.

3. Evaluate $\int_C |z|\,dz$, where C is left half of the unit circle $|z| = 1$ from $z = -i$ to $z = i$.

4. Using Cauchy Integral formula or otherwise show that $\int_C \frac{e^{-z}}{z^2}\,dz = -2\pi i$, where C is the ellipse $2x^2 + y^2 = 2$.

5. Expand $f(z) = \frac{1}{1-z}$ in the Laurent series valid for $|z - i| > \sqrt{2}$.

6. Using Residue theorem or otherwise show that $\int_C z\, exp\left(\frac{1}{z}\right) dz = \pi i$, where C is the unit circle $|z| = 1$.

7. Use residue to establish the integration formula $\int_{-\pi}^{\pi} \frac{d\theta}{1+sin^2\theta} = \pi\sqrt{2}$.

8. State and prove the orthogonality property of Chebyshev polynomials.

9. Tightly stretched flexible string has its ends fixed at $x = 0$ and $x = a$. At time $t = 0$, the string is given a shape defined by $f(x) = \mu x(a-x)$ where μ is a constant, and then released. Find the displacement of the string at any point x at time t.

10. Show that $\left(1-x^2\right)T_n'(x) = -nxT_n(x) + nT_{n-1}(x)$