

JAYPEE UNIVERSITY OF INFORMATION TECHNOLOGY, WAKNAGHAT

TEST-3 EXAMINATION- JUNE -2016

B. Tech (CE) II Semester

COURSE CODE: 10B11MA201

MAX. MARKS: 35

COURSE NAME: MATHEMATICS-II

COURSE CREDITS: 04

MAX. TIME: 2 HRS

Note: All questions are compulsory. Carrying of mobile phone during examinations will be treated as case of unfair means. The use of any calculator is not permitted in the examination.

1. Show that $|z|^2$ is differentiable at $z = 0$ but not analytic there. [3]
2. Determine the analytic function $f(z) = u + iv$, where $u = x(1 - y)$. [3]
3. Integrate $\int_C (z + 2\bar{z}) dz$ from $z = 0$ to $z = 1 + i$ along the curve
 $x = t, y = t^2, 0 \leq t \leq 1$. [3]
4. Evaluate the contour integral $\int_C \frac{\cos \pi z^2}{(z-1)(z-2)} dz$, where C is the circle $|z| = 3$ ccw. [4]
5. Expand the function $f(z) = \frac{1}{(z+1)(z+3)}$ in the Laurent series valid for
 $0 < |z + 1| < 2$. [4]
6. Let $f(z) = \frac{z-1}{(z+1)^2(z-2)}$. Find the residue of $f(z)$ at $z = -1$, and hence use residue theorem to evaluate $\int_C f(z) dz$, where C is the circle $|z - i| = 2$ ccw. [4]
7. Evaluate the real integral $\int_0^{2\pi} \frac{d\theta}{5+4\sin\theta}$. [4]
8. Solve the differential equation $y'' + y = 2x$ in series $\sum_{n=0}^{\infty} c_n x^n$. [5]
9. For Legendre polynomials show that

$$\int_{-1}^1 (1-x^2) P'_n(x) P'_m(x) dx = \begin{cases} 0 & \text{if } m \neq n \\ \frac{2n(n+1)}{(2n+1)} & \text{if } m = n \end{cases} \quad [5]$$