Jaypee University of Information Technology, Waknaghat (T-1 Examination, February 2016)

Course Title: Probability Theory and Random Processes

Program: B.Tech (ECE/CSE/IT)

Course Code: 10B11MA411 Semester: IV

Marks: 15 marks

Time: 1 hour

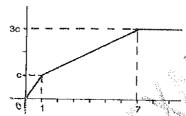
Note: Answer all the questions

1. $\Gamma(a)$ If $A \subset B$, $\mathbb{P}(A) = 1/4$, and $\mathbb{P}(B) = 1/3$, find $\mathbb{P}(A|B)$ and $\mathbb{P}(B|A)$.

- (b) If A and B are mutually exclusive events with $\mathbb{P}(A) = 0.4$ and $\mathbb{P}(B) = 0.2$, find $\mathbb{P}(\overline{A} \cap B)$ and $\mathbb{P}(\overline{A} \cup \overline{B})$. (2 Marks)
- 2. It is your birthday! Time to do some math. Mom or Dad have hidden a present for you, either upstairs or downstairs. With probability 0.6, the present was hidden by Mom. If Mom has hidden the present, it is upstairs with probability 0.7. Dad is equally likely to have hidden it upstairs or downstairs.

 (3 Marks)
 - (a) What is the probability that the present is upstairs?
 - (b) If it is upstairs, what is the probability that it was hidden by Dad?
- 3. Consider the following distribution function of a random variable:

(2 Marks)



- (a) Find the value of c
- (b) Write down the density function.
- 4. (a) Let X be a random variable with $\mathbb{P}(X = 1) = 0.70$, $\mathbb{P}(X = 10) = 0.20$, and $\mathbb{P}(X = 100) = 0.10$. Compute the moment generating function of X.
 - (b) Suppose that $\mathcal{M}_{\mathbf{X}}(\mathbf{t}) = 1 + \frac{t}{2} + \frac{t^2}{6} + \frac{t^3}{24} + \cdots$. Determine the variance of \mathbf{X} . (3 Marks)
- 5. Suppose that 3 balls are chosen without replacement from an urn consisting of 5 white and 8 red balls. Let X_i equal 1 if the i^{th} ball selected is white, and let it equal 0 otherwise. Give the joint probability mass function of X_1 and X_2 . (2 Marks)
- 6. Consider the joint density function of (X, Y):

(3 Marks)

$$f(x, y) = \begin{cases} \frac{6}{7}(x^2 + \frac{xy}{2}), & 0 < x < 1 \text{ and } 0 < y < 2 \\ 0, & \text{otherwise} \end{cases}$$

- (a) Find the marginal density function of X.
- (b) Find $\mathbb{P}(Y > 1/2 \mid X < 1/2)$.

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