Harinder Singh

JAYPEE UNIVERSITY OF INFORMATION TECHNOLOGY, WAKNAGHAT

TEST-3 EXAMINATION JUNE-2016

B.Tech (CS/ECE/IT) IV Semester

COURSE CODE: 10B11MA411 MAX. MARKS: 35

COURSE NAME: Probability Theory and Random Processes

COURSE CREDITS: 4 MAX. TIME: 2 HRS

Note: *All questions are compulsory. Carrying of mobile phone during examinations will be treated as case of unfair means.*

Q(1) An engineering company advertises a job in three papers, A, B and C. It is known that these papers attract undergraduate engineering readerships in the proportions 2: 3: 1. The probabilities that an engineering undergraduate sees and replies to the job advertisement in these papers are 0.002, 0.001 and 0.005 respectively. It is assumed that the undergraduate reads only one paper.

(a) If the engineering company receives only one reply to it advertisements, calculate the probability that the applicant has seen the job advertised in paper A.

(b) If the company receives two replies, what is the probability that both applicants saw the job advertised in paper A?

(3+2=5)

Q(2)

(a) Suppose that (T_1, T_2, T_3) is a sequence of independent random variables, and that T_i has the exponential distribution with rate parameter $\lambda_i = 2$ for each $i \in \{1,2,3\}$. Find the distribution function of $X = \max\{T_1, T_2, T_3\}$. Also find the probability density function of X.

(b) For the discrete random variable X, the probability distribution is given by:

$$P(X = x) = \begin{cases} kx & x = 1,2,3,4,5 \\ k(10 - x) & x = 6,7,8,9 \end{cases}$$

(i) Find the value of the constant k .

(ii) Determine E(X).

(iii) If $P(X \le c) \ge \frac{8}{25}$, find minimum value of c.

(2+3=5)

Q(3)

(a) Entry to a certain University is determined by a national test. The scores on this test are normally distributed with a mean of 525 and a standard deviation of 100. Tom wants to be admitted to this university and he knows that he must score better than at least 90% of the students who took the test. Tom takes the test and scores 585. Will he be admitted to this university?

(b) The length of similar components produced by a company is approximated by a normal distribution model with a mean of 5 cm and a standard deviation of 0.02 cm. If a component is chosen at random,

(i) What is the probability that the length of this component is between 4.98 and 5.02 cm?

(ii) What is the probability that the length of this component is between 4.96 and 5.04 cm?

Given Standard normal probabilities $\Phi(z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2}\, dz$:

z	0.0	0.5	1.0	1.5	2.0	2.5
$\Phi(z)$	0.5000	0.6915	0.8413	0.9332	0.9772	0.9938

(2+3=5)

Q(4) The time elapsed between the claims processed is modeled such that T_k represents the time elapsed between processing the $(k-1)^{th}$ and k^{th} claim where T_1 is the time until the first claim is processed, etc. Given $T_1, T_2, \ldots, T_k$ are mutually independent; and the pdf of T_k is exponential with mean time 5 hours.

(i) Calculate the probability that at least one claim will be processed in the next 5 hours.

(ii) What is the probability that at least 3 claims processed within 5 hours?

(iii) What is the probability that at least 6 hours will be waiting time for 3 claims? **(1+2+2=5)**

Q(5)

(a) Assume a random process $\{X(t)\}$ with four sample functions:

$$X(t,s_1) = \cos(t), \quad X(t,s_2) = -\cos(t), \quad X(t,s_3) = \sin(t), \quad X(t,s_4) = -\sin(t)$$

where s_1, s_2, s_3, s_4 are equally likely outcomes of a random experiment. Show that it's WSS process.

(b) Assume a random process $X(t) = \cos(t+\phi)$, where ϕ is a random variable following continuous uniform distribution on range $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. Check whether it is wide sense stationary process. **(2+3=5)**

Q(6)

(a) Let $X(t)$ be a Poisson process with parameter $\lambda = 5$. Find

(i) $E(X^2(2))$

(ii) $E[\{X(10) - X(4)\}^2]$

(b) A computer device can be either in a busy mode processing a task, or in an idle mode, where there are no tasks to process. Being in a busy mode, it can finish a task and enter and enter an idle mode any minute with the probability 0.2. Being in an idle mode, it receives a new task any minute with the probability 0.1 and enters a busy mode. The initial state is idle. Let X_n be the state of the device after n minutes.

(i) Find the transition probability matrix (TPM).

(ii) Find the steady-state distribution of X_n. **(2+3=5)**

Q(7) Consider the WSS Gaussian random process X(t) with the auto-correlation function $R_{xx}(\tau) = e^{-2|\tau|}$.

(a) For the given $R_{xx}(\tau)$:

(i) Find mean and variance of the random process X(t).

(ii) Further find the mean vector and covariance matrix for the random vector $\begin{bmatrix} X(4) \\ X(6) \end{bmatrix}$.

(b) Find the joint pdf of the random vector $\begin{bmatrix} X(8) \\ X(10) \end{bmatrix}$.

(c) Find the power spectral density function for the random process X(t). **(2+2+1=5)**

The End