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JAYPEE UNIVERSITY OF INFORMATION TECHNOLOGY, WAKNAGHAT

END TERM TEST

SUMMER SEMESTER - JUNE 2016

B.Tech- IV Semester

COURSE CODE: 10B11MA411 MAX. MARKS: 50

COURSE NAME: Probability Theory and Random Processes

COURSE CREDITS: 04 MAX. TIME: 2 Hrs

***Note:** All questions are compulsory. Carrying of mobile phone during examinations will be treated as case of unfair means. Each question carries equal marks 5.*

1. A conversation in a wireless ad-hoc network is severely disturbed by interference signals according to a Poisson process of rate $\lambda = 0.1$ per minute.

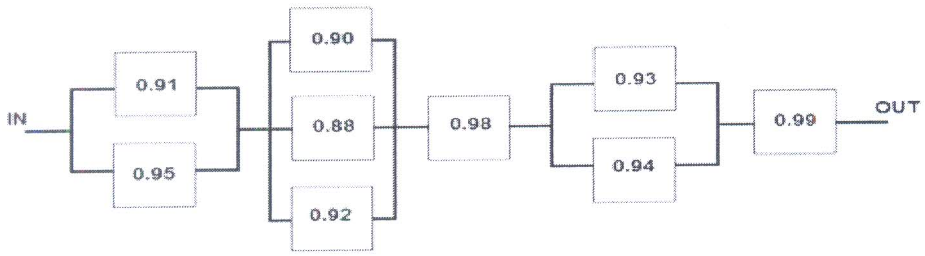
   (i) What is the probability that no interference signals occur within the first two minutes of the conversation?
   (ii) Find the probability for second interference signal will occur after three minutes of the conversation.
   (iii) Find the probability for third interference signal will occur after five minutes of the conversation.
   (iv) Given that the first two minutes are free of disturbing effects, what is the probability that in the next minute precisely 1 interfering signal disturbs the conversation?

2. The traffic on a one way street shown below may be satisfactorily described by a Poisson process with an average rate of arrival of 10 cars per minute. A driver is on a side street and is waiting to cross this main street. He will cross as soon as he finds a gap of 15 seconds.

   (i) Determine the probability that a gap will be longer than 15 seconds.
   (ii) What is the probability that the driver will cross at the fourth gap?
   (iii) Determine the mean number of gaps he has to wait until crossing the main road.
   (iv) What is the probability that he will cross within the first 4 gaps?

3. For given reliability function $R(t) = \frac{1}{(0.5\,t+1)^2}$; $t \geq 0$. Find
   (i) Failure density function
   (ii) Hazard rate function
   (iii) Mean time to failure (MTTF)

4. A block diagram representation of a system is shown below. Determine the overall system reliability.



5. Find E(X), V(X) and $P(2 < X < 5)$ where M.G.F. of the Normal random variable X is $e^{2(t+1)^2-2}$.

6. An automobile manufacturer introduces a new model that averages 27 miles per gallon in the city. A person who plans to purchase one of these new cars wrote the manufacturer for the details of the tests, and found out that the standard deviation is 4 miles per gallon. Assume that in-city mileage is approximately normally distributed.

   (i) What is the probability that the person will purchase a car that averages less than 23 miles per gallon for in-city driving?

   (ii) What is the probability that the person will purchase a car that averages between 23 and 29 miles per gallon for in-city driving?

7. Find the steady state probabilities for the given transition probability matrix

$$TPM = \begin{matrix} & S_1 & S_2 & S_3 \\ S_1 & 0.25 & 0.5 & 0.25 \\ S_2 & 0 & 0.5 & 0.5 \\ S_3 & 0.5 & 0 & 0.5 \end{matrix}$$

8. Calculate 2-step transition probability $p_{CR}^{(2)} = P(X_3 = R \mid X_1 = C)$ for a Markov chain with state space S = (R C) and transition probability matrix $P = \begin{matrix} & R & C \\ R & 0.4 & 0.6 \\ C & 0.25 & 0.75 \end{matrix}$.

   (i) Find $P(X_3 = R \mid X_1 = C, X_2 = R)$

   (ii) Find $P(X_5 = R, X_3 = C, X_1 = C, X_2 = R)$, where given is $P(X_1 = C) = 0.5$

9. Consider the WSS Gaussian random process X(t) with the auto-correlation function $R_{xx}(\tau) = e^{-\tau^2}$. For the given $R_{xx}(\tau)$:

   (i) Find mean and variance of the random process X(t).

   (ii) Further find the mean vector and covariance matrix for the random vector $\begin{bmatrix} X(4) \\ X(5) \end{bmatrix}$.

10. Find the power spectral density function for the WSS Gaussian random process X(t) with the auto-correlation function $R_{xx}(\tau) = 4\,e^{-|\tau|}$.

End

Given Standard normal probabilities $\Phi(z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2}\, dz$

z	0.0	0.5	1.0	1.5	2.0	2.5
$\Phi(z)$	0.5000	0.6915	0.8413	0.9332	0.9772	0.9938