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MID SEMESTER EXAMINATION-2015

M.Tech II SEMESTER

COURSE CODE: 10M11EC213 | MAX. MARKS: 30

COURSE NAME: INFORMATION AND CODING THEORY

COURSE CREDITS: 3 | MAX. TIME: 2 HRS

Note: All questions are compulsory. Answer each section at one place. Question in each section carry equal marks. Answers should be brief and to the point.

Section A (Marks: 6)

1. In discussing the noisy channel coding theorem, we had broken the coder in two parts, while we didn't do so in proving the converse. Why was it so?

2. Why is it that despite the optimum source code – the Huffman code – being known, new codes are still being developed?

3. Mutual information can be negative while self information cannot be negative. Why?

4. Can a function simultaneously be convex ∩ and convex ∪? Justify your answer.

5. Can the conditional information of an event x given y be more than the self information of x? Explain your answer without using mathematics.

6. In an experiment we have a channel with an input probability vector, A, a set of transition probability vectors, B, and an output probability vector, C. Does the channel capacity depend on A only, B only, C only, A and B, A and C, B and C, or all three?

Section B (Marks: 9)

1. Compute $g_{n(s)} = \sum_{y_n} P(y_n|0)^{1-s}\, P(y_n)^s$ for the binary symmetric channel with probability of error $P_e = 0.1$. Use this result to compute an upper bound on the probability of error for ML decoding with two codewords, $x_1 = 0000$ and $x_2 = 1111$.

2. (a) Determine which of these codes are uniquely decodable. (0, 01)(00,01,10,11)(110,11,10)(0,01,10)
   (b) State a rule for the lengths of instantaneous codes. Prove this is necessary and sufficient.

3. A has an unbiased coin which he tosses to determine if he should roll an unbiased dice (when the coin is heads) or a biased dice with P(1) = P(2) = 0.5, P(3) = 0.1, P(4) = 0.2 and P(5) = P(6) = 0.3. B has an unbiased dice. Both roll the dice. What is the mutual information of the coin tossed by A being heads, knowing that B has won?

Section C (Marks: 15)

1. A set of 8 messages with probabilities 0.2, 0.15, 0.15, 0.1, 0.1, 0.1, 0.1and 0.1 is to be is to be coded as a ternary prefix condition code. Construct two ~~optimum~~ coding schemes whose lengths have the same average but different variances. Why may one code be preferred to the other?

2. For the three channels whose transition probabilities are as follows:

   (i) P (0|0) =1, P (0|1) = P (1|2) = P (2|2) = 0.5, P (1|1) =P (2|1) = 0.25

   (ii) P (0|0) = P (1|1) = P (2|2) = 0.8, P (1|0) = P (2|1) =P (0|2) = 0.2

   (iii) P (0|0) = P (1|1) = 0.8, P (1|0) = P (0|1) = 0.2

Determine the channel capacity and the optimizing input probabilities.

3. (a) Prove that ln z <z – 1 when z ≠1.

   (b) Prove H (XY|Z) ≥ H (X|Z).

   (c) 4 bit binary representations of the numbers 0,3,5,6,9,10,12 1nd 15 are input to a binary symmetric channel with P_e = 0.1. How much information is provided about the message 0, given the first output bit?