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# Jaypee University of Information Technology, Waknaghat

## Test-III Examination - December 2021

### B.Tech (ECE)

Course Title: Probability Theory and Random Processes
Course Code: 18B11MA314
Semester: III

Max. Marks: 35 marks
Max. Time: 2 hours

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**Note:** Answer all the questions. Describe random variables along with range where applicable. Scientific calculators are allowed.

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1. You have built a robot that can detect defective items produced in your factory: if an item is defective, it is spotted with 98% probability by the robot; when an item is not defective, the robot will not signal any defect with 99% probability. You draw an item at random from a production lot in which 0.1% of items are defective. (3 Marks) [CO-1]

   (a) What is the probability that the robot signals a defective item?

   (b) If the robot tells us that the drawn item is defective, what is the probability that the robot is right?

2. Consider the CDF of a random variable $\mathbf{X}$: (4 Marks) [CO-2]

$$\mathbb{F}_{\mathbf{X}}(x) = \begin{cases} 0 & , \quad x < 0 \\ \frac{x}{8} + \frac{3x^2}{16} & , \quad 0 \leq x < 2 \\ 1 & , \quad x \geq 2 \end{cases}$$

   (a) Determine the density function of $\mathbf{X}$.

   (b) Find $\mathbb{P}(1 \leq \mathbf{X} \leq 1.5)$.

3. Consider the joint density of random variables $\mathbf{X}$ and $\mathbf{Y}$: (4 Marks) [CO-2]

$$\mathtt{f}(x,\ y) = \begin{cases} e^{-x} & , \quad 0 < y < x < \infty \\ 0 & , \quad \text{otherwise} \end{cases}$$

   Find $\mathbb{P}(\mathbf{Y} < 2 \mid \mathbf{X} = 5)$.

4. Consider the following problems. (4 Marks) [CO-3]

   (a) The time it takes to complete a service call to a mail-order computer company has exponential distribution with an average of 4 minutes. Within what time will 90% of the calls be completed?

   (b) A radioactive source emits particles according to a Poisson process with inter-arrival times (in minutes) distributed exponentially with parameter $\lambda = 2$. What is the probability that exactly one particle is emitted in the interval from 3 to 5 minutes?

5. Measure the height of your classmates. Ask them the height of their bench mate.

| | | | | | | |
|---|---|---|---|---|---|---|
| Height of classmates ($x$) | 5.5 | 5.2 | 5.4 | 5.5 | 5.6 | 5.8 |
| Height of benchmates ($y$) | 4.5 | 5.6 | 5.0 | 5.0 | 5.4 | 4.5 |

   Calculate the correlation coefficient of these two variables. (5 Marks) [CO-3]

6. Fit a parabola $y = a + bx + cx^2$ in *least squares* sense to the data: (5 Marks) [CO-4]

| x | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| y | −4 | −1 | 4 | 11 | 20 |

7. The average breaking strength of a material is required to exceed 150 psi. Past experience has indicated that the *standard deviation* of breaking strength is $\sigma = 3\ (psi)^2$. A random sample of four values are observed: 145, 147, 150, 153. (5 Marks) [CO-4]

   (a) State the hypotheses that you think should be tested in this experiment.

   (b) Use $z$-test to test these hypotheses with $\alpha = 0.05$. Take $z_{0.05} = 1.645$.

8. Number of defects per unit in a sample of 330 manufactured units was found as follows:

| No. of defects | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| No. of units | 214 | 92 | 20 | 3 | 1 |

   (a) Fit a Poisson distribution to the data.

   (b) Test the goodness-of-fit at 5% level of significance. (5 Marks) [CO-4]

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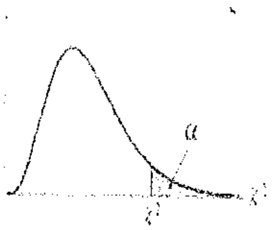
## Chi-Square Distribution:




Right tail

| Degrees of freedom | α | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 0.995 | 0.99 | 0.975 | 0.95 | 0.90 | 0.10 | 0.05 | 0.025 | 0.01 | 0.005 |
| 1 | — | — | 0.001 | 0.004 | 0.016 | 2.706 | 3.841 | 5.024 | 6.635 | 7.879 |
| 2 | 0.010 | 0.020 | 0.051 | 0.103 | 0.211 | 4.605 | 5.991 | 7.378 | 9.210 | 10.597 |
| 3 | 0.072 | 0.115 | 0.216 | 0.352 | 0.584 | 6.251 | 7.815 | 9.348 | 11.345 | 12.838 |
| 4 | 0.207 | 0.297 | 0.484 | 0.711 | 1.064 | 7.779 | 9.488 | 11.143 | 13.277 | 14.860 |
| 5 | 0.412 | 0.554 | 0.831 | 1.145 | 1.610 | 9.236 | 11.071 | 12.833 | 15.086 | 16.750 |
| 6 | 0.676 | 0.872 | 1.237 | 1.635 | 2.204 | 10.645 | 12.592 | 14.449 | 16.812 | 18.548 |
| 7 | 0.989 | 1.239 | 1.690 | 2.167 | 2.833 | 12.017 | 14.067 | 16.013 | 18.475 | 20.278 |
| 8 | 1.344 | 1.646 | 2.180 | 2.733 | 3.490 | 13.362 | 15.507 | 17.535 | 20.090 | 21.955 |
| 9 | 1.735 | 2.088 | 2.700 | 3.325 | 4.168 | 14.684 | 16.919 | 19.023 | 21.666 | 23.589 |
| 10 | 2.156 | 2.558 | 3.247 | 3.940 | 4.865 | 15.987 | 18.307 | 20.483 | 23.209 | 25.188 |