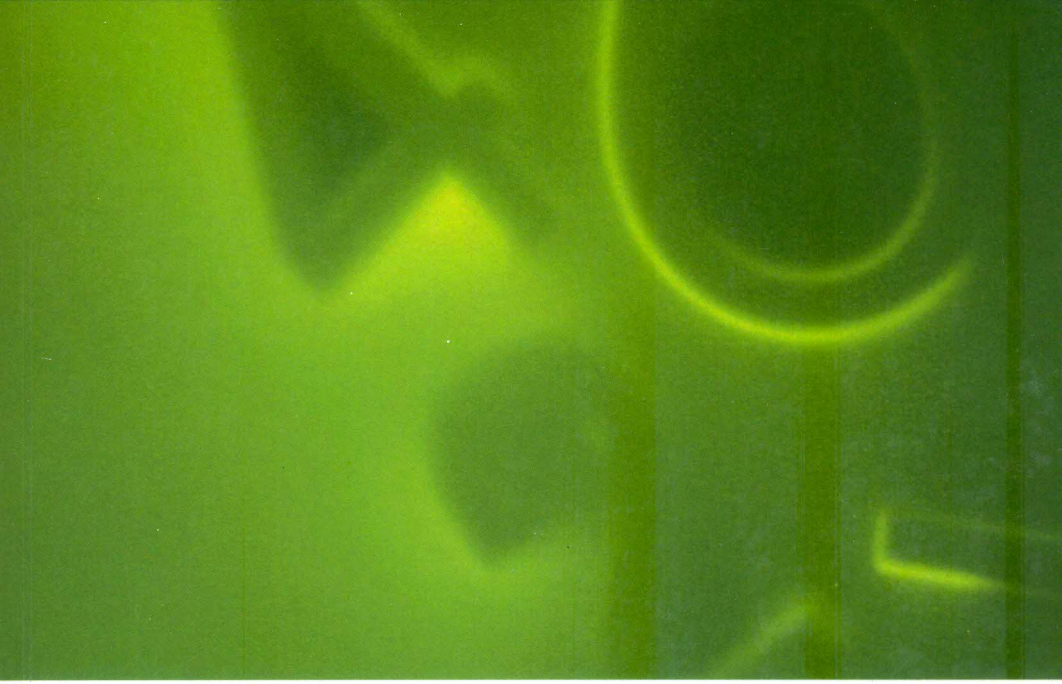

Rakesh Bajaj
D.S. Hooda
Harinder Singh

Fuzzy Sets and Information

A Generalized Approach

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Learning Resource Center
JUIT PUBLICATION
JUIT, Waknaghat, Solan (H.P.)

LAP LAMBERT Academic Publishing

Impressum/Imprint (nur für Deutschland/ only for Germany)
Bibliografische Information der Deutschen Nationalbibliothek: Die Deutsche Nationalbibliothek verzeichnet diese Publikation in der Deutschen Nationalbibliografie; detaillierte bibliografische Daten sind im Internet über http://dnb.d-nb.de abrufbar.
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Coverbild: www.ingimage.com

Verlag: LAP LAMBERT Academic Publishing AG & Co. KG
Dudweiler Landstr. 99, 66123 Saarbrücken, Deutschland
Telefon +49 681 3720-310, Telefax +49 681 3720-3109
Email: info@lap-publishing.com

Herstellung in Deutschland:
Schaltungsdienst Lange o.H.G., Berlin
Books on Demand GmbH, Norderstedt
Reha GmbH, Saarbrücken
Amazon Distribution GmbH, Leipzig
ISBN: 978-3-8383-5148-3

Imprint (only for USA, GB)
Bibliographic information published by the Deutsche Nationalbibliothek: The Deutsche Nationalbibliothek lists this publication in the Deutsche Nationalbibliografie; detailed bibliographic data are available in the Internet at http://dnb.d-nb.de.
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Publisher: LAP LAMBERT Academic Publishing AG & Co. KG
Dudweiler Landstr. 99, 66123 Saarbrücken, Germany
Phone +49 681 3720-310, Fax +49 681 3720-3109
Email: info@lap-publishing.com

Printed in the U.S.A.
Printed in the U.K. by (see last page)
ISBN: 978-3-8383-5148-3

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Fuzzy set theory has capability to describe the uncertain situations, containing ambiguity and vagueness. Fuzziness is found in our decision, in our thinking, in the way we process information, and particularly in our language.The objective of the book is to study fuzzy mathematics with applications and to characterize new measures of fuzzy information with their various generalizations. The fuzzy information measure deals with vague and ambiguous uncertainties. Large number of measures of entropy, directed divergence and symmetric divergence for probability distributions are known. Analogously, there is a set of variety of measures for fuzzy sets available. Cross-entropy measure is also known as the information discrepancy between two probability distributions. Derived from cross-entropy, fuzzy directed divergence measures the dissimilarity between two fuzzy sets. Various generalized measures of fuzzy information have been introduced. The concept of `useful' fuzzy information by attaching utility to the uncertainties of fuzziness and probabilities of randomness has been introduced.

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9 783838 351483 978-3-8383-5148-3